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## Abstract

For some public sector projects it is necessary to estimate the impact of the interest tax shield on the financial cost of capital. With debt financing, the value of the levered firm is increased by the present value of the interest tax shield. In this note, the present value of the interest tax shield is reconsidered. In finance textbooks, it is commonly assumed that the discount rate for the tax shield is d, the risk-free rate for debt. Here, it is shown that the correct discount rate for the tax shield is ρ, the required rate of return with all-equity financing.

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# Present Value of the Tax Shield in Project Appraisal: A Note

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Most textbooks on project appraisal have little or no discussion about the construction of the financial cashflow statement for projects, and if they do, the treatment is particularly brief. See Dinwiddy & Teal (1996), Boardman et al (1996), Brent (1996) and Ray (pg 133, 1984). A notable exception is the manual on cost-benefit analysis by Jenkins and Harberger (Chapter 3, 1997). Other references in cost-benefit analysis with discussion of the financial cashflow statement include Zerbe & Dively (Chapter 9, 1994), Sugden & Williams (Chapter 3, 1978), Gittinger (Chapter 3, 1982).

The reason for the neglect is understandable. For most public sector projects, the financial cashflow statement may not be relevant and consequently, the authors of the textbooks are eager to move directly to the treatment of the economic benefits and costs. However, for some public sector projects, the construction of the financial cashflow statement may be relevant. For example, in the case of a road project, the economic analysis may suggest that the economic benefits justify the cost and the project should be undertaken. In order to finance the project, we may wish to analyze the tariff structure which would be necessary to enable the road to be **financially** viable from the point of view of a private investor.

In such cases, it is important to construct both the financial and the economic cashflows. Furthermore, the financial cashflow itself may be constructed from different points of view depending on the number of different parties who have a stake in the project. Without the financial cashflow statements from alternative points of view, it would not be possible to conduct a proper risk analysis from alternative points of view. The construction of the financial and economic distributive cashflow statements, in conjunction with a distributive analysis derived from the financial and economic cashflow statements, provides a comprehensive analysis of the project. With the construction of the financial cashflow statements, it becomes necessary to estimate the relevant **financial** cost of capital which should be used to discount the respective cashflows; in particular, it is necessary to estimate the impact of the interest tax shield on the **financial** cost of capital because the value of the firm is increased by the present value of the tax shield.

In this note, I wish to reconsider the present value of the tax shield in the context of the typical textbook example with cashflows in perpetuity and constant debt-equity ratios; or if the debt-equity ratio is not constant, the financing is adjusted in order to maintain the constant debt-equity ratio. In practice, the annual cashflows, debt-equity ratios and tax shields for projects may follow more complex patterns and the typical textbook example may not be appropriate. Nevertheless, for ease of exposition the typical textbook example will be used. In the following discussion, I do not distinguish the project from the firm. I will use the phrase "cashflow of the firm" rather than "cashflow of the project". Technically, this is not quite correct but we can think of the project as stand-alone or alternatively, we can view the project as a 'carbon copy' of the risk profile of the firm. See Levy and Sarnat (pg 479, 1994)

It is generally accepted that the value of a levered firm B is equal to the value of the unlevered firm A plus the present value of the interest tax shield. That is, with financing, the value of the levered firm is increased by the present value of the interest tax shield. See Copeland and Weston (pg 443, 1988), Brealey and Myers (pg 476, 1996), Levy and Sarnat (pg 413, 1994) The annual free cashflow (FCF), in perpetuity, is equal to NOI*(1 - t), where NOI is the constant annual Net Operating Income and t is the constant corporate tax rate. If there is no debt financing, then the value of the unlevered firm A is equal to the net of tax NOI divided by the required rate of return $\rho$. That is,

$$(V^U) = \frac{NOI*(1 - t)}{\rho} \qquad \textbf{(1)}$$

where $(V^U)$ is the value of the unlevered firm A and $\rho$ is the required return for all-equity financing. See Copeland (pg 442, 1988)

Now consider the effect of financing on the value of the levered firm. Let D be the market value of the debt; d is the cost of the debt. Then the annual interest deduction is dD. The tax shield from the interest deduction is tdD. And

$$(V^L) = (V^U) + \text{Present Value of } tdD$$

$$= (V^U) + H \qquad \textbf{(2)}$$

where $(V^L)$ is the value of the levered firm B, and H is the present value of the tax shield.

The value of the levered firm B is equal to the value of the equity and the debt.

$$(V^L) = (E^L) + D \qquad \textbf{(3)}$$

where $(E^L)$ is the market value of the equity in firm B and D is the market value of the debt in firm B.

Let Z be the FCF for the equity holder who receives the net of tax operating income minus the payment for debt plus the tax shield. The free cashflow Z is also equal to e, the return on equity (with financing) times the value of the equity (with financing). Thus,

$$Z = e_0*(E^L) = NOI*(1 - t) - dD + tdD \qquad \textbf{(4)}$$

where $e_0$ is the return to equity with financing. Using equation 3 and equation 4, we obtain,

$$e_0 = \frac{NOI*(1 - t) - dD + tdD}{(V^L) - D} \qquad \textbf{(5)}$$

$$e_0 = \frac{NOI^*(1 - t) - dD + tdD}{(E^L)} \qquad \textbf{(6)}$$

We will show that in equilibrium, the following equation holds.

$$(V^L{}_0) = (V^U{}_0) + H, \qquad \textbf{(7)}$$

where $(V^U{}_0)$ is the equilibrium value of the unlevered firm and $(V^L{}_0)$ is the equilibrium value of the levered firm B. In equilibrium, $e_0$ is the return to equity with debt financing.

A question which naturally arises is: what is the correct discount rate for calculating the present value of the tax shield? It is commonly accepted that the cost of debt is the correct discount rate for the tax shield because the risk of the tax shield is the same as the risk of the debt. Brealey and Myers (pg 476, 1996) write:

> "The most common assumption is that the risk of the tax shields is the same as that of the interest payments generating them."

With this assumption, the present value of the interest tax shield H is equal to the annual tax shield tdD divided by the cost of debt d; equivalently, the present value of H is equal to the tax rate t times the value of debt D. The value of the levered firm B is given by:

$$(V^L) = (V^U) + \frac{tdD}{d} = (V^U) + tD \qquad \textbf{(8)}$$

In this note, I would like to determine the correct discount rate for calculating the present value of the tax shield and consequently, the value of the levered firm. It will be shown that the correct discount rate depends on the assumption about the amount of borrowing which is made to replicate or undo the leverage of the levered firm B. In section I, I will briefly review the arbitrage arguments which have been used to estimate the present value of the tax shields and consequently the value of the levered firm B. In section II and section III, I will present an algebraic formulation of the discussion in section one.

**Section I**

An arbitrage argument is used to determine the value of the levered firm B. We show that $(V^L) > (V^U_0) + H$ cannot hold; also, we show that $(V^L) < (V^U_0) + H$ cannot hold. Thus, the equality must hold, that is, $(V^L_0) = (V^U_0) + H$.

First, it is assumed that $(V^L) > (V^U_0) + H$. That is, the value of the levered firm is **higher** than the equilibrium value of the levered firm $(V^L_0)$. See equation 7. It will be shown that the inequality cannot hold and thus the value of the levered firm must be equal to the value of the unlevered firm plus the present value of the tax shield.

Let e be the return to equity for the levered firm in the case when $(V^L) > (V^U_0) + H$. Since $(V^L) > (V^L_0)$, we can conclude that $e < e_0$. because the same free cashflow for the equity holder is now based on a higher value for the levered firm B. We will show that $(V^L)$ is not an equilibrium value for firm B because a person who owns shares in firm B can increase his income by undertaking the following set of transactions.

Since the return to equity e is **lower** than $e_0$, there is an incentive for a person who owns shares in firm B to undertake the following transaction.

1. Sell the shares in the levered firm B and

2. Buy the shares in the unlevered firm A with the cash from the sale of the shares in firm B and some amount of **BORROWING**.

Initially, with the shares in firm B, there was a certain level of leverage based on the debt equity ratio of firm B. In order, to maintain the previous level of leverage, an **equivalent** amount will be borrowed.

It is shown that the inequality condition $(V^L) > (V^U{}_0) + H$ cannot hold because the above transaction maintains the same level of leverage and results in higher income, that is, the income after the transaction is higher than the income before the transaction.

Second, it is assumed that $(V^L) < (V^U{}_0) + H$. That is, the value of the levered firm is **lower** than the equilibrium value of the levered firm. Again, it will be shown that this inequality also cannot hold and thus the value of the levered firm must be equal to the value of the unlevered firm plus the present value of the tax shield. Let e be return to equity for the levered firm.

Since $(V^L) < (V^L{}_0)$, we can conclude that $e > e_0$. Again, we will show that $(V^L)$ is not an equilibrium value for firm B because a person who owns shares in firm A can maintain his level of leverage and increase his income by undertaking the following transaction. Since the return to equity e is **higher** than $e_0$, there is an incentive to

1. Sell the shares in the unlevered firm A and

2. Buy the shares in the levered firm B and **LEND** some amount.

Initially, with the shares in firm A, there was no leverage. After purchasing shares in the levered firm B, there is leverage. In order to undo the leverage from the purchase of shares in firm B, we can lend an **equivalent** amount to offset the leverage. We will use some of the cash from the sale of the shares in firm A to buy shares in firm B and the rest of the cash to undo the leverage. The amount of lending will depend on the debt-equity ratio in firm B. It is shown that the inequality condition $(V^L) < (V^U{}_0) + H$ also cannot hold because the above transaction maintains the level of leverage and results in higher income. That is, the income after the transaction is higher than the income before the transaction. Thus one is led to conclude that equality must hold in equilibrium, that is, $(V^L_0) = (V^U_0) + H$.

**Section II**

Consider the first possibility. Suppose $(V^L) > (V^U_0) + H$. And we own φ percent of the shares in firm B. Then the value of the shares in firm B is equal to $\varphi^*(E^L)$ where $(E^L)$ is the value of the equity in firm B. We will sell the shares in firm B. Using the cash from the sale of shares in firm B plus some borrowing, we will buy shares in firm A. The total amount of cash available, C.A., for the purchase of shares in firm A is:

$$\text{C.A.} = \varphi^*(E^L) + \varphi^*D = \varphi^*[(E^L) + D] = (V^L) \qquad \textbf{(9)}$$

The percent of debt in firm B = $D/(V^L)$. Let X be the amount that we will borrow. Recall that we will borrow an amount X such that the leverage before and after the transaction remains the same. We have to specify clearly the level of leverage before and after the transaction. Before the transaction, with the ownership of the shares in levered firm B, the level of leverage was $D/(V^L)$. After the transaction, we would like to maintain the same leverage ratio. That is, the amount of borrowing X to the total amount of cash available for investment in unlevered firm A should be equal to the ratio of the debt D to the total value of firm B.

$$\frac{X}{\text{C.A.}} = \frac{D}{(V^L)} \qquad \textbf{(10)}$$

From equation 9, we see that the amount of available cash invested in unlevered firm A is equal to the value of the shares in the levered firm B. Thus,

$$\frac{X}{\text{C.A.}} = \frac{X}{\varphi^*(V^L)} = \frac{D}{(V^L)} \qquad \textbf{(11)}$$

Based on line 11, we will borrow $X = \varphi^*D$. Next, we will compare the income before the transaction with the income after the transaction.

**Income BEFORE the transaction**

Before the transaction, we own φ percent of the shares in firm B which provides an equity return of e. The income from this is:

$$e^*\varphi^*(E^L) \quad \textbf{(12)}$$

**Income AFTER the transaction**

With the cash from the sale of shares in levered firm B and the borrowing, we will buy shares in the unlevered firm A.

The total amount of available cash is

$$C.\ A. = \varphi^*(E^L) + \varphi^*D \quad \textbf{(13)}$$

The gross income obtained by investing this amount of cash in the shares of unlevered firm A is equal to the total amount of available cash times the return from firm A.

$$\rho[\varphi^*(E^L) + \varphi^*D] \quad \textbf{(14)}$$

However, we also have pay $d^*\varphi^*D$ for the borrowing. Thus, the net income is

$$\rho[\varphi^*(E^L) + \varphi^*D] - d^*\varphi^*D \quad \textbf{(15)}$$

We have to show that with the same level of leverage before and after the transaction, the net income after the transaction in [Line (15)], is higher than the income before the transaction [Line (12)].

$$\rho[\varphi^*(E^L) + \varphi^*D] - d^*\varphi^*D > e^*\varphi^*(E^L) \quad \textbf{(16)}$$

Based on line 16, we can solve for e, the return to equity and determine the condition under which the inequality will hold.

Dividing both sides by φ and solving for e, we obtain

$$e*(E^L) > \rho(E^L) + (\rho - d)*D \quad \textbf{(17)}$$

From equation 4, we know that:

$$e*(E^L) = Z = NOI*(1 - t) - dD + tdD \quad \textbf{(18)}$$

Combining line 17 and line 18, we obtain

$$Z > \frac{\rho*Z}{e} + (\rho - d)*D \quad \textbf{(19)}$$

$$eZ > \rho*Z + (\rho - d)*De \quad \textbf{(20)}$$

$$\{Z - (\rho - d)*D\}e > \rho*Z \quad \textbf{(21)}$$

Adding and subtracting (ρ - d)*D*ρ, we obtain

$$\{Z - (\rho - d)*D\}e > \rho*Z - (\rho - d)*D*\rho + (\rho - d)*D*\rho \quad \textbf{(22)}$$

$$\{Z - (\rho - d)*D\}e > \{Z - (\rho - d)*D\}*\rho + (\rho - d)*D*\rho \quad \textbf{(23)}$$

Dividing by {Z - (ρ - d)*D}, we obtain,

$$e > \rho + \frac{(\rho - d)*D*\rho}{\{Z - (\rho - d)*D\}} \quad \textbf{(24)}$$

We can obtain an expression for {Z - (ρ - d)*D} by adding -(ρ - d)*D to both sides of equation 4.

$$Z - \rho D + dD = NOI*(1 - t) - dD + tdD + dD - \rho D \quad \textbf{(25)}$$

$$Z - (\rho - d)D = NOI*(1 - t) + tdD - \rho D \quad \textbf{(26)}$$

Substituting this expression in line 24, we obtain,

$$e > \rho + \underline{\qquad (\rho - d)*D*\rho \qquad} \quad \textbf{(27)}$$

$\{NOI*(1 - t) - \rho D + tdD\}$

Dividing the numerator and denominator of the second term by ρ, we obtain,

$$e > \rho + \frac{(\rho - d)*D}{\{NOI*(1 - t)/\rho - D + tdD/\rho\}} \qquad \textbf{(28)}$$

$$e > \rho + \frac{(\rho - d)*D}{\{E + tdD/\rho\}} \qquad \textbf{(29)}$$

As long as the inequality in line (29) holds, it will be possible to maintain the same level of leverage and increase the net income. Thus, in equilibrium, the inequality cannot hold. That is,

$$e = \rho + \frac{(\rho - d)*D}{\{E + tdD/\rho\}} \qquad \textbf{(30)}$$

Compare line 30 with line 17. We see that $(E^L)$, the value of the equity in the levered firm B $(E^L)$ is equal to $\{E + tdD/\rho\}$. Thus, with the debt financing the value of the equity has increased by the present value of the tax shield where the discount rate for the tax shield is ρ, the required rate of return on all-equity financing.

**Section III**

Now consider the second possibility. Suppose $(V^L) < (V^U_0) + H$. And we own φ percent of the shares in unlevered firm A. Then the value of the shares in unlevered firm A is equal to $\varphi*(E^U)$ where $(E^U)$ is the value of the equity in firm A. For the unlevered firm A, $(E^U)$ is equal to $(V^U)$. We will use some of the cash from the sale of the shares in unlevered firm A to buy shares in the levered firm B and use the rest of the cash to undo the leverage obtained by the purchase of shares in the levered firm B.

**Income BEFORE the transaction**

Before the transaction, we own φ percent of the shares in unlevered firm A which provides an equity return of ρ. The income from unlevered firm A is:

$$\rho*\varphi*(E^U) = \rho*\varphi*(V^U) \qquad \textbf{(31)}$$

**Income AFTER the transaction**

With the cash from the sale of shares in unlevered firm A, we will buy shares in levered firm B and lend some amount to undo the leverage. The total amount of cash we obtain from the sale of shares in unlevered firm A is

$$C.\ A. = \varphi*(E^U) = \varphi*(V^U) \qquad \textbf{(32)}$$

Line 32 also shows the cash available C.A. for purchasing the shares in the levered firm B. The percent of debt in firm B = $D/(V^L)$. Let X be the amount of money that we will lend out to undo the leverage. Again, we have to specify clearly the level of leverage before and after the transaction. Before the transaction, our level of leverage is zero. In order to maintain the same level of leverage, we need to maintain a zero level of leverage. I will lend out exactly the same amount of debt which I will acquire by purchasing the shares in levered firm B from the cash available from the sale of the shares in firm A. That is, the ratio of the lending X to the total value of the cash invested in the unlevered firm A should be equal to the ratio of the debt D to the value of the levered firm B.

$$\frac{X}{C.A.} = \frac{X}{\varphi*(V^U)} = \frac{D}{(V^L)} \qquad \textbf{(33)}$$

Solving for X, we obtain,

$$X = \varphi * D * \frac{(V^U)}{(V^L)} \quad \textbf{(34)}$$

The interest income which I will receive by lending out X is equal to

$$= d * \varphi * D * \frac{(V^U)}{(V^L)} \quad \textbf{(35)}$$

The amount of money which is available for investment in firm B

$$= \varphi * (V^U) - X \quad \textbf{(36)}$$

And the income which we will receive is

$$= e * [\varphi * (V^U) - X] \quad \textbf{(37)}$$

Thus, we have to show that, with the zero level of leverage before and after the transaction, the sum of the income after the transaction is higher than the income before the transaction.

$$e * [\varphi * (V^U) - X] + dX > \rho * \varphi * (V^U) \quad \textbf{(38)}$$

$$e * [\varphi * (V^U) - \varphi * D * \frac{(V^U)}{(V^L)}] + d * \varphi * D * \frac{(V^U)}{(V^L)} > \rho * \varphi * (V^U) \quad \textbf{(39)}$$

Multiplying both sides by $(V^L)/[\varphi*(V^U)]$ , we obtain

$$e * [(V^L) - D] + d * D > \rho(V^L) \quad \textbf{(40)}$$

Adding ρD to both sides, we obtain

$$e * [(V^L) - D] > \rho(V^L) - \rho D + \rho D - dD \quad \textbf{(41)}$$

$$e * (E^L) > \rho(E^L) + (\rho - d)D \quad \textbf{(42)}$$

Line 42 is similar to line 17. Using the same sequence of steps from line 18 to line 27, we can similarly conclude that

$$e > \rho + \frac{(\rho - d) * D}{\{E + tdD/\rho\}} \quad \textbf{(43)}$$

As long as the inequality in line (40) holds, it will be possible to maintain the same level of leverage and increase the net income. Thus, in equilibrium, the inequality cannot hold. That is,

$$e = \rho + \frac{(\rho - d)*D}{\{E + tdD/\rho\}} \qquad \textbf{(44)}$$

Again, compare line 44 with line 17. We see that ($E^L$), the value of the equity in the levered firm B ($E^L$) is equal to {E + tdD/ρ}.

**Conclusion**

Based on the above analysis, we can conclude that the value of the levered firm B is equal to the value of the unlevered firm A plus the value of the tax shield where the appropriate discount rate for the tax shield is ρ, the required return on equity for the all-equity firm.

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