

# Development Discussion Papers

## **Financial Discount Rates in Project Appraisal**

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In the financial appraisal of a project, the cashflow statements are constructed from two points of view: the Total Investment (TI) Point of View and the Equity Point of View. One of the most important issues is the estimation of the correct financial discount rates for the two points of view. In the presence of taxes, the benefit of the tax shield from the interest deduction may be excluded or included in the free cashflow (FCF) of the project. Depending on whether the tax shield is included or excluded, the formulas for the weighted average cost of capital (WACC) will be different. In this paper, using some basic ideas of valuation from corporate finance, the estimation of the financial discount rates for cashflows in perpetuity and single-period cashflows will be illustrated with simple numerical examples.

**JEL codes:** D61, G31, H43

**Keywords:** free cash flow (FCF), weighted average cost of capital (WACC), total investment point of view, equity point of view

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### INTRODUCTION

In the manual on cost-benefit analysis by Jenkins and Harberger (Chapter 3:12, 1997), it is stated that the construction of the financial cashflow statements should be conducted from two points of view:

1. The Total Investment (or Banker's) Point of View and
2. The Owner's (or Equity) Point of View.

The purpose of the Total Investment Point of View is to “determine the overall strength of the project.” See Jenkins & Harberger (Chapter 3:12, 1997). Also, see Bierman & Smidt (pg. 405, 1993). In **practical** project appraisal, the manual suggests that it would be useful to analyze a project by constructing the cashflow statements from the two points of view because “it allows the analyst to determine whether the parties involved will find it worthwhile to finance, join or execute the project”. See Jenkins & Harberger (Chapter 3:11, 1997). For a recent example of the application of this approach in project appraisal, see Jenkins & Lim (1988).

In practical terms, the relevance and need to construct and distinguish these two points of view in the process of project selection is unclear. That is, under what circumstances would we prefer to use the present value of the cashflow statement from the total investment point of view (CFS-TIP) rather than the present value of the cashflow statement from the equity point of view (CFS-EPV)? Jenkins & Harberger provide no discussion or guidance on the estimation of the appropriate discount rates for the two points of view. The conspicuous absence of a discussion on the **estimation** and **calculation** of the appropriate financial discount rates from the two points of

view is understandable. See Tham (1999). Within the traditional context of project appraisal, the relative importance of the economic opportunity cost of capital, as opposed to the financial cost of capital, has always been higher. However, in some cases, the financial cost of capital may be as important, if not more, in order to assess and ensure the financial sustainability of the project. Due to the lack of discussion in the manual, we do not know the explicit (or implicit) assumptions with respect to the relationship between the present value of the CFS-TIP and the present value of the CFS-EPV. For example, under what **conditions** would it be **reasonable** to assume that equality holds between the two points of view?

Jenkins & Harberger (Chapter 3:11, 1997) write:

“If a project is profitable from the viewpoint of a banker or the budget office but unprofitable to the owner, the project could face problems during implementation.”

This statement suggests that, in practice, inequality in the two present values is to be expected and could be a real possibility rather than the rare exception. However, the statement raises many questions. If in fact there is inequality in the present values, what is the source of the inequality? The above statement does not even hint at a possible reason for the divergence in the two present values. What is the meaning or interpretation of the two present values?

The interpretation of the two points of view is particularly problematic when the present values have opposite signs. The meaning or practical significance of this divergence for project selection is not explained nor is it grounded in any theory of cashflow valuation. If in fact, the inequality holds, then it is conceivable that the present value in one point of view is positive, while the present value in the other point of view is negative or vice versa. In project selection, when

would it be desirable to prefer one present value over the other (if at all) or do both present values have to be positive in order for a project to be selected?

The interpretation of the discrepancy between the (expected) present values in the two points of view is even more serious when Monte Carlo simulation is conducted on the cashflows statements because the variances of the two present values will be different. Consequently, the risk profiles of the cashflows from the two points of view will be different. Even with the same expected NPVs from the two points of view, the variances of the NPV from the two points of view would be different; the interpretation of the risk profiles will be even more difficult if the expected values of the NPV from the two points of view are **substantially** different.

The objective of this paper is to apply some ideas from the literature in corporate finance to elucidate the calculation of appropriate financial discount rates in practical project appraisal. The Cashflow Statement from the Total Investment Point of View (CFS-TIPV) is equivalent to the free cashflow (FCF) in corporate finance which is defined as the “after-tax free cashflow available for payment to creditors and shareholders.” See Copeland & Weston (pg. 440, 1988). However, we have to be careful to specify whether the CFS-TIPV (or equivalently the FCF) includes or excludes the present value of the tax shield that arises from the interest deduction with debt financing. The standard results of the models from corporate finance, if one were to accept the stringent assumptions underlying the models, would suggest that the present value from the two points of view are **necessarily** equal (in the absence of taxes).

At the outset, it is very important to acknowledge that the standard assumptions in corporate finance are very stringent and thus there is a legitimate question about the relevance of such perfect models to practical project appraisal. It is possible that many practitioners would consider such an application of principles from corporate finance to project appraisal to be

inappropriate. Such reservations on the part of practitioners are fully justified. A cursory perusal of the assumptions which would have to hold in the Modigliani & Miller (M & M) and Capital Asset Pricing Model (CAPM) worlds would persuade many readers that even in developed countries, most, if not all, of the assumptions are **seriously** violated in practice. The violations are particularly acute in the practice of project appraisal in developing countries with capital markets which are, at present, far from perfect and will be far from perfect in the foreseeable future. In other words, the M & M world or the CAPM world are ideal situations and may not correspond to the real world in any meaningful sense. Nevertheless, these ideas are extremely important and relevant.

The basic concepts and conclusions from the models in corporate finance with applications in project appraisal can be briefly summarized as follows.

1. We need to distinguish the return to equity with no-debt financing  $\rho$ , and the return to equity with debt financing  $e$ .
2. In the absence of taxes, financing does not affect the value of the firm or project.
3. The cashflow from the equity point of view with debt financing (CFS-EPV) is more risky than the cashflow from the equity point of view with no financing (CFS-AEPV).
4. In the presence of taxes, the value of the levered firm is higher than the value of the unlevered by the present value of the tax shield. However, a complete analysis suggests that it may be reasonable to assume that the overall effect of taxes is close to zero. See Benninga (pg. 257 & 259, 1997)
5. There are two ways to account for the increase in value from the tax shield. We can either lower the Weighted Average Cost of Capital (WACC) or include the present value of the tax shield in the cashflow statement. **In terms of valuation, both methods are equivalent.** See line 18 and line 27 for further details on the WACC.
6. With financing, the return to equity  $e$  is a positive function of the debt-equity ratio, that is, the higher the debt equity ratio  $D/E$ , the higher the return to equity  $e$ . See line 26.

I believe that the application of these concepts from corporate finance to the estimation of financial discount rates in practical project appraisal is very relevant and can provide a useful baseline for judging the results derived from other models with explicit **assumptions** that are closer to the real world. After understanding the calculations of the financial discount rates in the perfect world where M & M's theories and CAPM hold, we can begin to relax the assumptions and make serious contributions to practical project selection in the imperfect world that is perhaps marginally more characteristic of developing countries compared to developed countries.

In section 1, I will briefly introduce and discuss the two points of view in the absence of taxes. In Section 2, I will introduce the impact of taxes and review the formulas which are widely accepted in corporate finance for the two polar cases: cashflows of projects in perpetuity and projects with single period cashflows. See Miles & Ezzell (pg. 720, 1980). I will not derive or discuss the meanings of the formulas. Typically, the formulas assume that the cashflows are in perpetuity and the debt equity ratio is constant and the analysts assume that the formulas for perpetuity are good approximations for **finite** cashflows.

In Section 3, I will use a simple numerical example to illustrate the application of the formulas to cashflows in perpetuity. In Section 4, I will apply the same formulas to a single-period example and compare the results with the results from Section 3. Even though it is not technically correct, in the following discussion I will use the terms "firm" and "project" interchangeably.

## **SECTION I: Two Points of View**

A simple example would illustrate the difference between the two points of view in the financial analysis. Suppose there is a single-period project which requires an investment of \$1,000 at the end of year 0 and provides a return of \$1,200 at the end of year 1. For now, we will assume

that the inflation rate is zero and there are no taxes. Later we will examine the impact of taxes.

The CFS-TIPV for the simple project is shown below.

**Table 1.1: Cashflow Statement, Total Investment Point of View (CFS-TIPV)**

End of year>>	0	1
Revenues	0	1,200
Investment	1,000	0
<b>NCF (TIPV)</b>	<b>-1,000</b>	<b>1,200</b>

$$\text{The rate of return from the TIPV} = \frac{(1,200 - 1,000)}{1,000} = \mathbf{20.00\%}. \quad (1)$$

Now if there was no debt financing for this project, the CFS-TIPV would apply to the equity holder, that is, the equity holder would invest \$1,000 at the end of year 0 and receive \$1,200 at the end year 1.

**Table 1.2: Cashflow Statement, All-Equity Point of View (CFS-AEPV)**

End of year>>	0	1
Revenues	0	1,200
Investment	1,000	0
<b>NCF (AEPV)</b>	<b>-1,000</b>	<b>1,200</b>

Thus, in this special case with no taxes, the CFS-AEPV will be identical with the CFS-TIPV. Compare Table 1.1 and Table 1.2. We will see later that with taxes, there will be a divergence between the CFS-AEPV and the CFS-TIPV.

Suppose the minimum required return on all-equity financing  $\rho$  is 20%. Then this project would be acceptable. In this special case, for simplicity, the value of  $\rho$  was chosen to make the NPV of the CFS-AEPV at  $\rho$  to be zero.

The PV in year 0 of the CFS-AEPV in year 1 is

$$= \frac{1,200}{1 + 20\%} = \mathbf{1,000.00} \quad (2)$$

The NPV in year 0 of the CFS-AEPV is

$$= \frac{1,200}{1 + 20\%} - 1,000 = \mathbf{0.00} \quad (3)$$

Later, we consider an example where the NPV is positive. See line 21. Next, we will consider the effect of debt financing on the construction of the cashflow statements from the two points of view.

### **Debt financing**

Suppose, to finance the project, we borrow 40% of the investment cost at an interest rate of 8%.

$$\text{Debt (as a percent of initial investment)} = 40\% \quad (4)$$

$$\text{Equity (as a percent of initial investment)} = 1 - 40\% = \mathbf{60\%} \quad (5)$$

$$\text{Debt-Equity Ratio} = \frac{40\%}{60\%} = \mathbf{0.667} \quad (6)$$

$$\text{Amount of debt, } D = 40\% * 1000 = \mathbf{400.00} \quad (7)$$

At the end of year 1, the principal plus the interest accrued will be repaid.

$$\text{Repayment in year 1} = D * (1 + d) = \mathbf{432.00} \quad (8)$$

The loan schedule is shown below.

**Table 1.3: Loan Schedule**

End of year>>	0	1
Repayment	0	-432
Loan	400	0
Financing @ 8%	<b>400</b>	<b>-432</b>

We can obtain the Cashflow Statement from the Equity Point of View (CFS-EPV) by combining the CFS-TIPV with the cashflow of the loan schedule. The CFS-EPV is shown below.

**Table 1.4: Cashflow Statement, Equity Point of View (CFS-EPV)**

End of year>>	0	1
NCF (TIPV)	-1,000	1,200
Financing	400	-432
NCF (EPV)	<b>-600</b>	<b>768</b>

The rate of return (ROR) for the CFS-EPV,

$$e = (768 - 600)/600 = \mathbf{28.00\%}. \quad (9)$$

With 40% financing, the equity holder invests only 600 at the end of year 0 and receives 768 at the end of year 1.

Note the difference between CFS-AEPV and CFS-EPV (Compare **Table 1.2** and **Table 1.4**). With debt financing, the risk is higher for the equity holder and thus the return must be higher to compensate for the higher risk. See Levy & Sarnat (pg. 376, 1994)

The critical question is: what should be the appropriate financial discount for the cashflow statements from the two points of view. We will apply M & M's theory which asserts that, in the absence of taxes, the value of the levered firm should be equal to the value of the unlevered firm. That is, financing does not affect valuation.

$$\begin{aligned} \text{Value of unlevered firm, } (V^{UL}) \\ = (V^L), \text{ Value of levered firm} \end{aligned} \quad (10)$$

In turn, the value of the levered firm is equal to the value of the equity ( $E^L$ ) and the value of the debt  $D$ .

$$(V^L) = (E^L) + D \quad (11)$$

In other words, in the absence of taxes, the correct discount rate for the CFS-TIPV is equal to the required return on all-equity financing, namely  $\rho$ .

The discount rate  $w$  for the CFS-TIPV is also commonly known as the Weighted Average Cost of Capital (WACC). The present value of the cashflow statement from the all-equity point of view at  $\rho$  is equal to the present value of the cashflow statement from the total investment point of view discounted at the WACC. See equation 12 below.

$$PV[\text{CFS-AEPV}]_{@ \rho} = PV[\text{CFS-TIP}]_{@ w} \quad (12)$$

In present value terms, we can also write the following equivalent expression for line 11. The present value of the CFS-TIP is equal to the present value of the equity in the levered firm plus the present value of the debt.

$$PV[\text{CFS-TIP}]_{@ w} = PV[\text{CFS-EPV}]_{@ e} + PV[\text{CFS-Loan}]_{@ d} \quad (13)$$

Combining equations (12) and (13), we can write the following expression,

$$PV[\text{CFS-AEPV}]_{@ \rho} = PV[\text{CFS-EPV}]_{@ e} + PV[\text{CFS-Loan}]_{@ d} \quad (14)$$

The present value of the cashflow statement with all-equity financing is equal to the present value of the equity cashflow plus the present value of the loan.

We can verify the above identity in the context of the simple example above. Compare line 2 and line 15.

$$PV[\text{Cashflow}]^{\text{TIP}}_{@ \rho} = \frac{1,200}{1 + 20\%} = \mathbf{1,000.00} \quad (15)$$

The present value of the CFS-TIP, discounted at  $\rho$  is 1,000; as shown below in line 16 and line 17, the present value of the CFS-EPV at  $e$ , is 600, and the present value of the loan repayment at  $d$  is 400, respectively.

$$PV[\text{Cashflow}]^{\text{Equity}}_{@ e} = \frac{768}{1 + 28\%} = 600.00 \quad (16)$$

$$PV[\text{Cashflow}]^{\text{Loan}}_{@ d} = \frac{432}{1 + 8\%} = 400.00 \quad (17)$$

What these calculations show is a simple but powerful idea. It has been shown numerically that the discount rate  $w$  for CFS-TIP is a weighted average of the return on equity and the cost of debt; the value of weights are based on the relative values of the debt and equity. It is easy to confirm the above statements with algebra. We simply provide a simple numerical confirmation of the fact that the WACC is equal to  $\rho$ . See equation 1 and Table 1.2.

$$\begin{aligned} w &= \text{Percent Equity} * \text{Return on Equity} + \text{Percent Debt} * \text{Cost of Debt} \\ &= \%E * e + \%D * d \\ &= 60\% * 28\% + 40\% * 8\% = \mathbf{16.80\%} + \mathbf{3.20\%} = \mathbf{20.00\%} \quad (18) \end{aligned}$$

We can also use a well-known expression to calculate the value of  $e$ . Note that the following expression for  $e$  in line 19 is a function of the return to all-equity financing  $\rho$ , the cost of debt  $d$ , and the debt equity ratio  $D/E$ .

$$\begin{aligned}
 e &= \rho + (\rho - d) \frac{D}{E} \\
 &= 20\% + (20\% - 8\%) \frac{40\%}{60\%} \\
 &= 20\% + \mathbf{8.00\%} = \mathbf{28.00\%} \qquad (19)
 \end{aligned}$$

Again, this calculation of the value of  $e$  in line 19 matches the previous calculation. See Table 1.4 and compare line 19 with line 9.

### Cashflow with positive NPV

In the previous example, we had chosen specific numerical values to ensure that the NPV of the CFS-AEPV was zero. See line 3. In practice, it would be rare to find a project whose NPV was exactly zero. Instead suppose that the annual revenues was 1,250. Then the cashflow statement would be as shown in Table 1.5.

**Table 1.5: Cashflow Statement, All-Equity Point of View (CFS-AEPV)**

End of year>>	0	1
Revenues	0	1,250
Investment	1,000	0
NCF (AEPV)	<b>-1,000</b>	<b>1,250</b>

The rate of return (ROR) for the CFS-AEPV,

$$= \frac{(1,250 - 1,000)}{1,000} = \mathbf{25.00\%}. \qquad (20)$$

In this case, the rate of return of the CFS-AEPV is greater than  $\rho$ . Compare line 20 with line 1.

In year 0, the NPV of the CFS-AEPV is

$$= \frac{1,250}{1 + 20\%} - 1,000 = \mathbf{41.67} \quad (21)$$

Compare line 21 with line 3. In year 0, the PV of the CFS-AEPV in year 1

$$= \frac{1,250}{1 + 20\%} = \mathbf{1,041.67} \quad (22)$$

Compare line 22 with line 2. Since the NPV of the CFS-AEPV is positive in line 21, we have to make an adjustment in the calculation of the WACC. In the calculation of the total value of the debt plus equity, we have to use the total value of 1,041,67 in line 22 and recalculate the percentage of debt and equity.

$$\text{Debt (as a percent of total value)} = \frac{400}{1,041.67} = \mathbf{38.40\%} \quad (23)$$

$$\text{Equity (as a percent of total value)} = 1 - \mathbf{38.40\%} = \mathbf{61.60\%} \quad (24)$$

$$\text{Debt-Equity Ratio} = \frac{38.40\%}{61.60\%} = \mathbf{0.623} \quad (25)$$

Thus, the percentage of debt as a percentage of the total value is 38.40% and **not** 40%.

Compare line 23, line 24, and line 25 with line 4, line 5, and line 6 respectively.

Also, we have to recalculate the return to equity  $e$  with the new debt-equity ratio.

$$\begin{aligned} e &= \rho + (\rho - d) \frac{D}{E} \\ &= 20\% + (20\% - 8\%) \frac{38.40\%}{61.60\%} \\ &= 20\% + \mathbf{7.48\%} = \mathbf{27.48\%} \end{aligned} \quad (26)$$

Using the revised debt and equity ratios and the return to equity, we can calculate the Weighted Average Cost of Capital (WACC).

$$\begin{aligned}
 w &= \text{Percent Equity} * \text{Return on Equity} + \text{Percent Debt} * \text{Cost of Debt} \\
 &= \%E * e + \%D * d \\
 &= 61.60\% * 27.48\% + 38.40\% * 8\% \\
 &= \mathbf{16.93\% + 3.07\% = 20.00\%} \qquad (27)
 \end{aligned}$$

As expected, the WACC in line 27 is equal to the WACC in line 18 and the value of  $\rho$  in line 1. Using the value of WACC, we can find the PV of the CFS-TIP in year 1

$$= \frac{1,250}{1 + 20\%} = \mathbf{1,041.67} \qquad (28)$$

The results of the two cases are summarized in the following table. Case 1 is the original numerical example with zero NPV (See line 3) and Case 2 is the numerical example with positive NPV (See line 21) In practice, it is the rare case where the NPV is zero; however, as shown here, with a positive NPV, we simply have to adjust the debt and equity ratios by using the value shown in line 28.

**Table 1.6: Summary results for case 1 (NPV = 0) and case 2 (NPV > 0)**

	<b>Case1</b>	<b>Case1</b>	<b>Case2</b>	<b>Case2</b>
	<b>No debt</b>	<b>With debt</b>	<b>No debt</b>	<b>With debt</b>
%Debt	0%	40%	0%	38.40%
%Equity	100%	60%	100%	61.60%
D/E Ratio	0.00	0.667	0.00	0.623
Equity Return	20%	28%	25%	27.48%
WACC	*****	20%	*****	20%
Net Present Value	0.00	0.00	41.67	41.67
PV of Cashflow	1,000.00	1,000.00	1,041.67	1,041.67

In summary, the above numerical example demonstrates that the present value of the CFS-TIP at the WACC is equal to the present value of the CFS-EPV (Table 1.4) plus the present value of the loan repayment (Table 1.3).

In this section, we had assumed that there were no taxes. In the next section, we will examine the complications that arise in the presence of taxes. With taxes, there are similar formulas for the calculation of the WACC.

## **SECTION II: Impact of taxes**

In the previous example we did not have taxes. With taxes, some adjustments have to be made in the above formulas. Because of the tax benefit from the interest deduction, it can be shown that the value of the levered firm is equal to the value of the unlevered firm plus the present value of the tax shield. See any standard corporate finance textbook. In particular, see Copeland & Weston (pg. 442, 1988)

$$\begin{aligned} \text{Value of levered firm} \\ &= \text{Value of unlevered firm} + \text{Present Value of Tax Shield} \end{aligned}$$

$$(V^L) = (V^U) + PV(\text{Tax Shield}) \quad (29)$$

$$= (E^L) + D \quad (30)$$

Compare line 30 with line 11. With debt financing, the value of the equity is increased by the present value of the tax shield.

It is commonly assumed that the appropriate discount rate for the tax shield is  $d$ , the cost of debt. See Copeland & Weston (pg. 442, 1988) and Brealey & Myers (pg. 476, ) With taxes, there are two equivalent ways of expressing the CFS-TIP. In constructing the Total Investment Cashflow, we can either exclude or include the effect of the tax shield in the CFS-TIP. If we do

not include the tax shield in the cashflow, then the Total Investment Cashflow would be identical to the all-equity cashflow CFS-AEPV. Thus, we will use the following abbreviations.

CFS-AEPV = Cashflow Statement without the tax shield.

CFS-TIP = Cashflow Statement with the tax shield

The value of the WACC that is used for discounting the Total Investment Cashflow will depend on whether the tax shield is or excluded or included. See Levy & Sarnat (pg. 488, 1994) If the tax shield is excluded, then in the construction of the income statement, the interest deduction will be excluded in order to determine the tax liability **as if** there was no financing. If the tax shield is included, then in the construction of the income statement, the interest deduction will be included in order to determine the correct tax liability.

**Method 1:** Excluding the tax shield and using CFS-AEPV

Line 31 and line 32 show the equations for calculating w and e in the traditional approach. Since the cashflow statement does not include the tax shield, the value of the tax shield is taken into account in the WACC by multiplying the cost of debt d by the factor (1 - t).

$$w = \text{Percent Equity} * \text{Return on Equity} + \text{Percent Debt} * \text{Cost of Debt} * (1 - \text{tax rate})$$

$$w = \%E * e + \%D * d * (1 - t) \tag{31}$$

$$e = \rho + (1 - t) * (\rho - d) * \frac{D}{E} \tag{32}$$

Compare line 31 with line 18; and compare line 32 with line 19. The value of the equity E includes the present value of the tax shield and thus the debt and equity ratios will be different

from the original values. Similarly, in the expression for the return to equity in line 32, there is the additional factor (1-t).

**Method 2:** Including the tax shield and using CFS-TIP

In this case the cashflow statement includes the value of the tax shield. Consequently, unlike in Method 1, there is no need to adjust the expressions for the calculation of the WACC and the return to equity  $e$ . The appropriate formulas are shown in line 33 and line 34.

$w = \text{Percent Equity} * \text{Return on Equity} + \text{Percent Debt} * \text{Cost of Debt}$

$$w = \%E * e + \%D * d \quad (33)$$

$$e = \rho + (1 - t) (\rho - d) * \frac{D}{E} \quad (34)$$

Compare line 31 with line 33 and compare line 32 with line 34. Note that line 33 is the same as line 18. There is no difference between line 32 and line 34. Again, note that the value of the equity  $E$  in line 31 to line 34 includes the present value of the tax shield and thus the debt and equity ratios will be different from the original values. For further details, see line 42 and line 43 below.

In terms of valuation, it makes no difference whether method 1 or method 2 is used. In the past, method 1 was preferred because it was computationally simpler. See Levy & Sarnat (pg. 489, 1944). However, these days, computation time is probably not a relevant consideration.

**SECTION III: Cashflows in perpetuity**

In this section, we will apply the above formulas to a specific numerical example. We will continue to assume that the inflation rate is zero. The corporate tax rate is 40%. The framework

in this section, with all the standard assumptions, is based on Copeland & Weston (pg. 442, 1988). We will compare the cashflow statements with and without debt financing.

Assume that a simple project generates annual revenues of 15,000 in perpetuity. The annual operating costs are 3,000. To maintain the constant annual revenues, the annual reinvestment will be equal to the annual depreciation which is assumed to be 2,000. With the reinvestment assumption, the annual Net Operating Income (NOI) after tax will be equal to the annual cashflow from the Equity Point of View. See Copeland & Weston (pg. 441, 1988)

The detailed income statement for the project is shown below.

**Table 3.1: Income Statement (in perpetuity)**

	Yr>>	0	1	2
Revenues			15,000.00	====>>>>
Operating Cost			3,000.00	====>>>>
Depreciation			2,000.00	====>>>>
Gross Margin			<b>10,000.00</b>	====>>>>
Interest Deduction			00.00	====>>>>
Net Profit before taxes			<b>10,000.00</b>	====>>>>
Taxes			4,000.00	====>>>>
Net Profit after taxes			<b>6,000.00</b>	====>>>>

The Net Operating Income (NOI) before tax is 10,000; the value of the tax payments is 4,000 and the Net Operating Income (NOI) after tax is \$6,000. It is assumed that the required return on equity with all-equity financing  $\rho$  is 6%. The detailed cashflow statement is shown below. The annual free cashflow is 6,000. As noted before, the annual cashflow in perpetuity is equal to the Net Operating Income (NOI) after tax in perpetuity.

**Table 3.2: Cashflow Statement, Total Investment Point of View/Equity Point of View**

	Yr>>	0	1	2
Revenues			15,000.00	====>>>>
Total Inflows			<b>15,000.00</b>	====>>>>
				====>>>>
Investment			2,000.00	====>>>>
Operating Cost			3,000.00	====>>>>
Total Outflows			<b>5,000.00</b>	====>>>>
Net Cashflow before tax			<b>10,000.00</b>	====>>>>
Taxes			<b>4,000.00</b>	====>>>>
Net Cashflow after tax			<b>6,000.00</b>	====>>>>

The value of the unlevered cashflow is shown below.

$$\begin{aligned}
 (V^{UL}) &= \frac{NOI \cdot (1 - t)}{\rho} = \frac{FCF}{\rho} = \frac{10,000 \cdot (1 - 40\%)}{6\%} \\
 &= \frac{\mathbf{6,000.00}}{6\%} = \mathbf{100,000.00} \qquad (35)
 \end{aligned}$$

Thus, based on the annual cashflow of 6,000 in perpetuity, the value of the unlevered firm is 100,000.

### Debt Financing

Next we consider the cashflow statement with debt financing. We will assume that the debt as a percent of the total value of the unlevered firm is 30%; thus, the value of the debt is 30,000. The interest rate on the debt  $d$  is 5% and the annual interest payment on the debt

$$= D \cdot d = 30,000 \cdot 5\% = \mathbf{1,500.00}. \qquad (36)$$

The annual tax savings is equal to  $t \cdot d \cdot D$

$$= t \cdot d \cdot D = 40\% \cdot 5\% \cdot 30,000 = \mathbf{600.00}. \qquad (37)$$

The present value of the tax shield in perpetuity =  $t \cdot D$

$$= 40\% \cdot 30,000 = \mathbf{12,000.00}. \qquad (38)$$

The income statement, with the interest deduction, is shown below.

**Table 3.3: Income Statement**

	Yr>>	0	1	2
Revenues			15,000.00	====>>>>
Operating Cost			3,000.00	====>>>>
Depreciation			2,000.00	====>>>>
Gross Margin			<b>10,000.00</b>	====>>>>
Interest Deduction			1,500.00	====>>>>
Net Profit before taxes			<b>8,500.00</b>	====>>>>
Taxes			<b>3,400.00</b>	====>>>>
Net Profit after taxes			<b>5,100.00</b>	====>>>>

The value of the tax payments is 3,400 and the net profit after taxes is 5,100. Previously, the tax payments was 4,000. The difference in the values of the two tax payments in Table 3.2 and Table 3.3 is equal to the tax savings from the deduction of the interest payments.

The Total Investment Cashflows, without and with the tax shield, are shown below in Table 3.4 and Table 3.5 respectively.

**Table 3.4: Cashflow Statement, Total Investment Point of View, without tax shield**

	Yr>>	0	1	2
Net Cashflow before tax			10,000.00	====>>>>
Taxes			<b>4,000.00</b>	====>>>>
Net Cashflow after tax			<b>6,000.00</b>	====>>>>

If we exclude the value of the tax shield in the cashflow, then the cashflow in year 1 in the CFS-TIPV will be 6,000. See Table 3.4. Again, recall that the cashflow will be identical to CFS-AEPV. In this case, the tax liability is 4,000 and the FCF available for distribution to the debt holders and the equity holders is 6,000.

If we include the value of the tax shield in the cashflow, then the cashflow in year 1 in the CSF-TIPV will be 6,600 as shown below.

**Table 3.5: Cashflow Statement, Total Investment Point of View, with tax shield**

	Yr>>	0	1	2
Net Cashflow before tax			10,000.00	====>>>>
Taxes			<b>3,400.00</b>	====>>>>
Net Cashflow after tax			<b>6,600.00</b>	====>>>>

The difference in the cashflows in Table 3.4 and Table 3.5 is the present value of the tax shield.

### Calculation of the value of the levered firm

We know that the value of the levered firm is equal to the value of the unlevered firm plus the present value of the tax shield.

$$\begin{aligned}
 (V^L) &= (V^{UL}) + \text{Present Value of tax shield} \\
 &= 100,000 + \mathbf{12,000.0} = \mathbf{112,000.0} \qquad (39)
 \end{aligned}$$

In this case, the value of the tax shield is equal to 12,000 and thus the value of the levered firm increases from 100,000 to 112,000 due to the tax shield. Equivalently, the value of the equity in the levered firm increases by the present value of the tax shield to 82,000.

$$(E^L) = (V^L) - D = 112,000 - 30,000 = \mathbf{82,000.0} \qquad (40)$$

The amount of debt as a percent of the value of the unlevered firm was 30%; however, with the increase in the value of the levered firm from the tax shield, the amount of debt as a percent of the value of the levered firm decreases from 30% to 26.8%.

$$\text{Debt (as a percent of total value)} = \frac{30,000}{112,000} = \mathbf{26.78571\%} \qquad (41)$$

$$\text{Similarly, the new debt equity ratio} = \frac{30,000}{82,000} = \mathbf{0.366} \qquad (42)$$

The annual FCF available for distribution to the debt holders and the equity holders is 6,600. The Cashflow Statement from the Equity Point of View is shown below.

**Table 3.6: Cashflow Statement, Equity Point of View**

	Yr>>	0	1	2
NCF, TIP, after taxes			6,600.0	====>>>>
Interest Payment			1,500.0	====>>>>
NCF, Equity			<b>5,100.00</b>	====>>>>

Note that the FCF is equal to the net profit after taxes because we have assumed that the annual reinvestment is equal to the annual depreciation. See Table 3.3.

### Return to equity

After paying the annual interest payments of 1,500, the annual FCF to the equity holder is 5,100. Based on an equity value of 82,000, the return to equity (with debt financing) is

$$e = \frac{5,100}{82,000} = 6.21951\% \quad (43)$$

Alternatively, we could use the formulas which we had presented before. See line 32 and line 34.

The rate of return to the equity owner

$$e = \rho + (1 - t) * (\rho - d) * \frac{D}{E}$$

$$= 6\% + (1 - 40\%) * (6\% - 5\%) * \frac{30,000}{82,000} = 6.21951\% \quad (44)$$

If there was no tax and the FCF remained the **same**, then the return to equity would be

$$e = \rho + (\rho - d) * \frac{D}{E}$$

$$= 6\% + (6\% - 5\%) * \frac{30,000}{70,000} = 6.42857\% \quad (45)$$

See line 19. Compare the rate of return to equity in line 44 and line 45. With the tax shield, the return to equity is reduced from 6.429% to 6.22%. Alternatively, if there were no taxes, and assuming that the FCF remained the **same**, the return to the equity would be

$$\frac{(6,000 - 1,500)}{70,000} = \mathbf{6.429\%} \quad (46)$$

which is the same as the answer in line 45.

### Calculation of the WACC

We will calculate the WACC in two different ways and use them to estimate the value of the levered firm. As expected both values of the WACC will give the same answer.

#### WACC with Method 1

$$\begin{aligned} w_1 &= \text{Percent Debt} * \text{Cost of Debt} * (1 - t) + \text{Percent Equity} * \text{Cost of Equity} \\ &= \%D * d * (1 - t) + \%E * e \\ &= 26.78571\% * 5\% * (1 - 40\%) + 73.21429\% * 6.21951\% \\ &= \mathbf{0.80357\% + 4.55357\% = 5.35714\%} \quad (47) \end{aligned}$$

For the debt and equity ratios, see line 42. For the return to equity, see line 44. We can use the value of the WACC in line 47 to calculate the value of the levered firm.

$$\text{PV}[\text{Cashflow}]^{\text{TP}}_{@ w_1} = \frac{6,000}{5.35714\%} = 112,000 \quad (48)$$

Compare line 48 with line 39. They are the same.

## WACC with Method 2

$$\begin{aligned}w_2 &= \text{Percent Debt} * \text{Cost of Debt} + \text{Percent Equity} * \text{Cost of Equity} \\ &= \%D * d + \%E * e \\ &= 26.78571\% * 5\% + 73.21429\% * 6.21951\% \\ &= \mathbf{1.33929\% + 4.55357\% = 5.89286\%}\end{aligned}\tag{49}$$

$$PV[\text{Cashflow}]^{\text{TIP}}_{@ w_2} = \frac{6,600}{5.89286\%} = 112,000\tag{50}$$

As expected, both the valuations of the levered firm with the different WACC values give the same result. Again, compare line 50 with line 39.

Also, compare the present values in line 48 and line 50. If we exclude the tax shield in the FCF, then the correct WACC is 5.34% from line 47; alternatively, if we include the tax shield in the FCF, then the correct WACC is 5.89% from line 49.

We can also verify the following identity for the value of the levered firm.

$$(V^L) = (E^L) + D\tag{51}$$

$$\begin{aligned}PV[\text{Cashflow}]^{\text{TIP}}_{@ w_1} \\ &= PV[\text{Cashflow}]^{\text{Equity}}_{@ e} + PV[\text{Cashflow}]^{\text{Loan}}_{@ d}\end{aligned}\tag{52}$$

$$PV[\text{Cashflow}]^{\text{Equity}}_{@ e} = \frac{5,100}{6.21951\%} = \mathbf{82,000.0}\tag{53}$$

$$PV[\text{Cashflow}]^{\text{Loan}}_{@ d} = \frac{1,500}{5\%} = 30,000.0\tag{54}$$

Line 52 is equal to the sum of line 53 and line 54.

For comparative purposes, we can also calculate the WACC in the absence of taxes using the return to equity in line 45.

### WACC with no taxes

$$\begin{aligned}
 w &= \text{Percent Debt} * \text{Cost of Debt} + \text{Percent Equity} * \text{Cost of Equity} \\
 &= \%D * d + \%E * e \\
 &= 30\% * 5\% + 70\% * 6.42857\% \\
 &= \mathbf{1.50\% + 4.50\% = 6.00\%} \qquad (55)
 \end{aligned}$$

The results of the above analyses, with and without taxes, are summarized in the following table.

**Table 3.7: Summary of the example with and without tax**

	No Tax	With Tax
FCF	6,000	6,000
Cost of Debt	5%	5%
Amount of Debt	30,000	30,000
PV of tax shield	0	12,000
Debt (as % of $V^{UL}$ )	30%	30%
Debt (as % of $V^L$ )	30%	<b>26.79%</b>
Debt (as % of $E^L$ )	<b>42.86%</b>	<b>36.59%</b>
Value of Equity	70,000	82,000
Return to Equity	<b>6.429%</b>	<b>6.220%</b>
Value of firm	100,000	112,000
WACC (1)	6%	<b>5.357%</b>
WACC (2)	6%	<b>5.893%</b>

For practical project appraisal, we can summarize the above discussion as follows. If we exclude the tax shield in the cashflow statement, then to find the value of the levered firm, we discount the Total Investment Cashflow (CFS-AEPV) at 5.357%; if we include the tax shield in the cashflow statement, then to find the value of the levered firm, we discount the Total Investment Cashflow (CFS-TIP) at 5.893%. It does not matter which value of WACC is used; both WACCs used with the appropriate cashflow statements will give the correct value for the levered firm. See Table 3.4 and Table 3.5.

#### **SECTION IV: Single Period Cashflow**

In this section, we will apply the same formulas in line 31 through line 34 to a single-period project. We will continue to assume that the inflation rate is zero and the corporate tax rate is 40%. We will following the pattern of the analysis in Section III and compare the cashflow statements with and without debt financing.

Assume a simple project that generates revenues of 2,800 at the end of year 1. The initial investment required at the end of year 0 is 2,000. The annual operating cost is 500. The value of the depreciation is equal to the value of the initial investment. The detailed income statement is shown below.

**Table 4.1: Income Statement**

	Yr>>	0	1	2
Revenues			2,800.00	
Operating Cost			500.00	
Depreciation			2,000.00	
Gross Margin			<b>300.00</b>	
Interest Deduction			00.00	
Net Profit before taxes			<b>300.00</b>	
Taxes			<b>60.00</b>	
Net Profit after taxes			<b>240.00</b>	

At the end of year 1, the Gross Margin is 300. For the moment, we are assuming that there is no financing and thus the interest deduction is zero. The tax liability is equal to the Gross Margin times the tax rate times

$$= 300 * 20\% = \mathbf{60.00}. \quad (56)$$

The Net Profit after taxes is \$240. We assume that  $\rho$ , the required rate of return with all-equity financing is 12%. The Cashflow Statement from the Equity Point of View is shown below.

The free cashflow (FCF) in year 1 is equal to the net income plus depreciation.

$$\text{FCF} = \text{Depreciation} + \text{Net Profit after Taxes} \quad (57)$$

**Table 4.2: Cashflow Statement, Total Investment Point of View/Equity Point of View**

	Yr>>	0	1	2
Revenues			2,800.00	
Total Inflows			<b>2,800.00</b>	
Investment		2,000.00		
Op Cost			500.00	
Total Outflows		2,000.00	500.00	
Net Cashflow before tax		-2,000.00	<b>2,300.00</b>	
Taxes			60.00	
Net Cashflow after tax		-2,000.00	<b>2,240.00</b>	
NPV @ $\rho = 12.0\%$		<b>0.00</b>		
IRR		<b>12.00%</b>		

The rate of return on this project is 12% which is equal to the required return on equity of 12% for a project with all-equity financing.

$$\text{The present value of the FCF} = \frac{2,240}{1 + 12\%} = \mathbf{2,000.00} \quad (58)$$

Thus, based on the FCF of 2,240 at the end of year 1, the value of the unlevered firm is 2,000. As shown below, the NPV of the project is zero. As explained above, for simplicity we

have assumed a project with zero NPV. If the NPV of the project was positive, some minor adjustments would have to be made in the formulas. See the explanations for Table 1.5 in Section 1.

$$\text{The NPV of the FCF} = -2,000 + \frac{2,240}{1 + 12\%} = \mathbf{0.00} \quad (59)$$

Next we consider the cashflow statement with debt financing. We will assume that the debt as a percent of the total value of the unlevered firm is 60%; thus, the value of the debt is 1,200. The interest rate on the debt  $d$  is 8% and at the end of year 1, the interest payment on the debt

$$= D*d = 1,200*8\% = \mathbf{96.00}. \quad (60)$$

The loan schedule is shown below.

**Table 4.3: Loan Schedule**

	Yr>>	0	1	2
Beg Balance			1,200.00	0
Interest			96.00	
Payment			1,296.00	
End balance		1,200.00	0.00	
Financing		1,200.00	-1,296.00	
NPV @ = 8.0 %		<b>0.00</b>		
IRR		<b>8.00%</b>		

At the end of year 1, the total repayment for the loan, principal plus interest, is 1,296. The income statement, with the interest deduction, is shown below.

**Table 4.4: Income Statement**

	Yr>>	0	1	2
Revenues			2,800.00	
Operating Cost			500.00	
Depreciation			2,000.00	
Gross Margin			<b>300.00</b>	
Interest Deduction			<b>96.00</b>	
Net Profit before taxes			<b>204.00</b>	
Taxes			<b>40.80</b>	
Net Profit after taxes			<b>163.20</b>	

$$\text{The interest payment in year 1} = 8\% * 1,200 = \mathbf{96.00}. \quad (61)$$

In year 1, the full principal plus the interest accrued will be repaid. The value of the tax shield in year 1 is equal to the tax rate\*interest payments

$$= 20\% * 96 = \mathbf{19.20} \quad (62)$$

The amount of the tax payments is 40.80 and the net profits after tax is 163.20. With debt financing, the tax payments are reduced by the value of the tax shield from 60 to 40.80. Compare Table 4.2 and Table 4.4.

In constructing the FCF or TIP cashflow statement, there are two ways of showing the effect of the tax shield. See line 31 and line 33.

**Method 1.** In the traditional approach, we construct the after-tax FCF without the tax shield and adjust the discount rate. See line 31 and Table 4.5 below.

**Table 4.5: Cashflow Statement without the tax shield**

	Yr>>	0	1
Net Cashflow before tax		-2,000.00	2,300.00
Taxes, without financing			60.00
Net Cashflow after tax		-2,000.00	2,240.00
NPV @ $\rho = 12.0\%$		0.00	
IRR		12.00%	

This FCF in Table 4.5 is identical to the previous Equity cashflow in Table 4.2. In year 1, the FCF before tax is 2,300. The tax liability is 60, and thus the FCF after-tax is 2,240.

**Method 2.** Alternatively, we can include the tax shield in the construction of the after-tax FCF and use an appropriate discount rate.

**Table 4.6: Cashflow Statement with the tax shield**

	Yr>>	0	1
Net Cashflow before tax		-2,000.00	2,300.00
Taxes, with financing			40.80
Net Cashflow after tax		-2,000.00	2,259.20
NPV @ $\rho = 12.0\%$		17.143	
IRR		12.96%	

In year 1, the FCF before tax is 2,300 which is the same as in Table 4.5. With the tax shield from the financing, the tax liability is only 40.80, and thus the FCF after-tax in Table 4.6 is **2,259.20**. This cashflow is higher than the value in Table 4.5 by the value of the tax shield. Below, both of these approaches will be used to calculate value of the levered firm.

### Calculation of the value of the levered firm

We know that the value of the levered firm is equal to the value of the unlevered firm plus the present value of the tax shield.

$$(V^L) = (V^{UL}) + \text{Present Value of tax shield} \quad (63)$$

It is a common assumption that the tax shield should be discounted at the cost of debt, namely  $d$ . See Brealey & Myers (pg 476, 1996)

In year 1, the tax shield = the tax rate\*interest payments =  $tdD$

$$= 20\% * 96 = \mathbf{19.20} \quad (64)$$

Thus, the value of the levered firm is given by the following expression. Compare line 63 with line 39.

$$(V^L) = (V^{UL}) + \frac{tdD}{1 + d} \quad (65)$$

In year 0, the present value of the tax shield in year 0

$$\begin{aligned} &= \frac{TdD}{1 + d} \\ &= \frac{20\% * 96}{1 + 8\%} = \mathbf{17.7778} \end{aligned} \quad (66)$$

$$\begin{aligned} (V^L) &= (V^{UL}) + \text{Present Value of tax shield} \\ &= 2,000 + \mathbf{17.7778} = \mathbf{2,017.78} \end{aligned} \quad (67)$$

In this case, the value of the tax shield is equal to 17.78 and thus the value of the levered firm increases from 2,000 to 2,017.78 due to the tax shield.

$$(E^L) = (V^L) - D = 2,017.78 - 1,200 = \mathbf{817.78} \quad (68)$$

Equivalently, the value of the equity in the levered firm increases by the present value of the tax shield to 817.78.

The amount of debt as a percent of the value of the **unlevered** firm was 60%; however, with the increase in the value of the levered firm from the tax shield, the amount of debt as a percent of the value of the **levered** firm decreases from 60% to 59.5%.

$$\text{Debt (as a percent of total value)} = \frac{1,200}{2,017.78} = \mathbf{59.47130\%} \quad (69)$$

$$\text{The new debt equity ratio} = \frac{1,200}{817.78} = \mathbf{1.467} \quad (70)$$

Compare line 69 and line 70 with line 41 and line 42 respectively.

The annual FCF available for distribution to the debt holders and the equity holders is 2,259.20. The Cashflow Statement from the Equity Point of View is shown below.

Table 4.7 shows the equity cashflow statement with the tax shield.

**Table 4.7: Cashflow Statement, Equity Point of View, with Tax Shield**

	Yr>>	0	1	2
NCF, TIP, after taxes		-2,000.0	2,259.20	0.0
Financing		1,200.0	-1,296.00	0.0
NCF, Equity		-800.0	<b>963.20</b>	0.0
NPV @ 18.0 %		<b>16.271</b>		
IRR		<b>20.40%</b>		

Thus, the equity contribution at the end of year 0 (without taking into account the present value of the tax shield) is 800 and the FCF in year 1 is 963.20.

### **Different ways to calculate the return to equity**

There are many different ways to calculate the return on equity.

1. Use the original value of equity, without including the present value of the tax shield.
2. Use the perpetuity formula from corporate finance.
3. Increase the value of equity in year 0 by the present value of the tax shield.

Based on the initial equity value of 800 (without including the present value of the tax savings from the tax shield), the rate of return to the equity owner is

$$e = \frac{(963.20 - 800)}{800} = \mathbf{20.40\%} \quad (71)$$

We can also calculate the return to equity in two other ways. The first way is to use the formula in line 32. Again, note that the value of the equity E has been increased by the present value of the tax shield. See line 68.

The rate of return to the equity owner  $e = \rho + (1 - t) \cdot (\rho - d) \cdot \frac{D}{E}$

$$= 12\% + (1 - 20\%) \cdot (12\% - 8\%) \cdot \frac{1,200}{817.78} = \mathbf{16.69564\%} \quad (72)$$

We can also calculate the return to equity as follows. For the equity owner, the FCF in year 1 is 963.20 (See Table 4.7) and the value of the equity at the end of year 0 (including the tax shield) is 817.78. See line 68. Thus, the return to equity is

$$e = \frac{963.20 - 817.78}{817.78} = \mathbf{17.78229\%} \quad (73)$$

There is a small inexplicable discrepancy between the two approaches. Compare the returns in line 72 and line 73.

### Return to Equity with no taxes

If there were no taxes and the FCF remained the **same**, then the return to equity would be 18%, as shown in Table 4.8 below.

**Table 4.8: Cashflow Statement, Equity Point of View, No tax Shield**

	Yr>>	0	1	2
NCF, TIP, after taxes		-2,000.0	2,240.0	0.0
Financing		1,200.0	-1,296.0	0.0
NCF, Equity		-800.0	944.00	0.0
NPV @ = 18.0 %		0.000		
IRR		18.00%		

We can also use the formula in line 19. The rate of return to the equity owner e.

$$e = \rho + (\rho - d) \cdot \frac{D}{E}$$

$$= 12\% + (12\% - 8\%) \cdot \frac{1,200}{800} = \mathbf{18.00\%} \quad (74)$$

## Calculation of the Weighted Average Cost of Capital (WACC)

We will calculate the WACC in two different ways and use them to estimate the value of the levered firm.

### WACC with Method 1

$$\begin{aligned}w_1 &= \text{Percent Debt} * \text{Cost of Debt} * (1 - t) + \text{Percent Equity} * \text{Cost of Equity} \\&= \%D * d * (1 - t) + \%E * e \\&= 59.47130\% * 8\% * (1 - 20\%) + 40.52870\% * 16.69564\% \\&= \mathbf{3.80616\% + 6.76653\% = 10.57269\%} \quad (75)\end{aligned}$$

We can use this value of the WACC to calculate the value of the levered firm.

$$PV[\text{Cashflow}]^{\text{TIP}}_{@ w_1} = \frac{2,240}{(1 + 10.57269\%)} = \mathbf{2,025.817} \quad (76)$$

### WACC with Method 2

$$\begin{aligned}w_2 &= \text{Percent Debt} * \text{Cost of Debt} + \text{Percent Equity} * \text{Cost of Equity} \\&= \%D * d + \%E * e \\&= \%D * d + \%E * e \\&= 59.47130\% * 8\% + 40.52870\% * 16.69564\% \\&= \mathbf{4.75770\% + 6.76653\% = 11.52423\%} \quad (77)\end{aligned}$$

$$PV[\text{Cashflow}]^{\text{TIP}}_{@ w_2} = \frac{2,259.20}{1 + 11.52423\%} = \mathbf{2,025.748} \quad (78)$$

We expect both values of the WACC to give the same answer for the value of the firm. However, there is a **small** discrepancy of approximately one percentage point. Compare line 75 with line 77. In present value terms, the difference is very small. Compare line 76 with line 78. The results are summarized in the Table 4.9 below.

**Table 4.9: Comparison of Method 1 and Method 2.**

	<b>Method 1</b>	<b>Method 2</b>	<b>Difference</b>
<b>WACC</b>	10.57%	11.52%	<b>-0.95%</b>
<b>Value of Levered Firm</b>	2,025.817	2,025.748	<b>0.069</b>
<b>Valued of equity (levered)</b>	<b>825.82</b>	<b>825.75</b>	

Due to the discrepancy in the WACC, there is a difference in the value of the firm. Compare the WACCs in line 75 and line 77. Also, compare the value of the levered firm in line 76 and line 78 with the value of equity in line 67 which was derived by adding the present value of the tax shield to the original value of the equity. Again, there is an inexplicable discrepancy in the values.

### **WACC in the absence of taxes**

Also, we can calculate the WACC in the absence of taxes using the return to equity in line 74.

$$\begin{aligned}
 w &= \text{Percent Debt} * \text{Cost of Debt} + \text{Percent Equity} * \text{Cost of Equity} \\
 &= \%D * d + \%E * e \\
 &= 60\% * 8\% + 40\% * 18\% \\
 &= \mathbf{4.80\% + 7.20\% = 12.00\%} \qquad (79)
 \end{aligned}$$

### Verification of the value of the levered firm

We can also verify the following identity.

$$(V^L) = (E^L) + D \quad (80)$$

$$\begin{aligned} PV[\text{Cashflow}]^{\text{TIP}}_{@ \text{ w1}} \\ = PV[\text{Cashflow}]^{\text{Equity}}_{@ \text{ e}} + PV[\text{Cashflow}]^{\text{Loan}}_{@ \text{ d}} \end{aligned} \quad (81)$$

Using the return to equity in line 72, we obtain the value of equity as follows.

$$PV[\text{Cashflow}]^{\text{Equity}}_{@ \text{ e}} = \frac{963.20}{1 + 16.69564\%} = \mathbf{825.395} \quad (82)$$

Alternatively, we can use the return to equity in line 73.

$$PV[\text{Cashflow}]^{\text{Equity}}_{@ \text{ e}} = \frac{963.20}{1 + 17.78229\%} = \mathbf{817.780} \quad (83)$$

$$PV[\text{Cashflow}]^{\text{Loan}}_{@ \text{ d}} = \frac{1,296}{1 + 8\%} = \mathbf{1,200.00} \quad (84)$$

Due to the differences in the values of the return to equity, there are differences in the value of the equity. Compare line 82 and line 83.

The results of the above analysis, with and without taxes, are summarized in the following table.

**Table 4.10: Summary of the results**

	<b>No Tax</b>		<b>With Tax</b>
Cost of Debt	8%		8%
Amount of Debt	1,200		1,200
PV of tax shield	0		<b>17.7778</b>
Debt (as % of $V^{UL}$ )	60%		60%
Debt (as % of $V^L$ )	60%		<b>59.47130%</b>
Debt (as % of $E^L$ )	<b>150.0%</b>		<b>146.739%</b>
Value of Equity	800		817.78
Return to Equity (1)	<b>18.00%</b>		<b>16.69564%</b>
Return to Equity (2)	*****		<b>17.78229%</b>
Value of Equity (1)	*****		825.395
Value of Equity (2)	*****		817.78
Value of firm	2,000		2,017.78
WACC (1)	12%		10.57269%
WACC (2)	12%		11.52423%
Levered value with WACC1	*****		2,025.817
Levered value with WACC2	*****		2,025.748

## Conclusion

In the above analysis, I analyzed the two extreme cases: perpetuity cashflows and single period cashflow. In particular, with the simple numerical examples, I illustrated how the ideas from corporate finance can be usefully applied in the construction of financial cashflow statements in applied project appraisal. Also, I have shown that the value of the WACC depends on whether the tax shield is excluded or included from the FCF; however, the results are the same from both the different forms of the WACC.

In practice, neither of these two extreme cases prevail. Thus, in practice, simplifications have to be made to apply the ideas developed above. One of the most common assumption is to assume that the debt-equity ratio is constant for the life of the project. And in addition, we assume that for the limited range of values for the debt-equity ratio, the return to equity  $e$  is constant.

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