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**Multiperiod Financial Discount Rates**
**in Project Appraisal**

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## Abstract

The typical assumption about cash flows in perpetuity is not appropriate in practical project appraisal because the length of project life is always finite. In this paper, I present and discuss the calculation of multiperiod financial discount rates for a project with a finite life and no personal taxes. The impact of corporate taxes and inflation will also be included in the analysis.

With financing, there are two possible profiles. First, we may assume that a constant debt-equity ratio is maintained during the life of the project. The loan schedule is constructed to keep the debt-equity ratio constant for the life of the project. Second, the loan schedule may be fixed. In this case, the debt-equity ratio changes over the life of the project. By explicitly calculating the appropriate discount rate for each period, we can explicitly model the cashflow statement from the Total Investment Point of View (CFS-TIP) and the Equity Point of View (CFS-EPV). It is not necessary to assume that the debt-equity ratio is constant and the cash flows are in perpetuity.

**JEL Codes**: D61, G31, H43

**Key words:** Multiperiod WACC, cost of capital, project appraisal, impact of inflation.

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*On request, another version of the paper with illustrative numerical examples may be obtained from the author. Constructive feedback and critical comments are welcome. The author may be contacted by email at **JTham@hiid.vnn.vn**.

# Multiperiod Financial Discount Rates in Project Appraisal

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## INTRODUCTION

In standard books on corporate finance and project appraisal, the formulas for calculating the Weighted Average Cost of Capital (WACC) are based on three key assumptions. First, the formulas are given for cash flows in perpetuity. Second, it is assumed that the debt-equity ratio is constant for the life of the project. For example, see Copeland & Weston (pg 442, 1988), Levy and Sarnat (pg 404, 1994) and Brealey and Myers (pg 517, 1996). Third, it is assumed that the risk of the tax shield is the same as the risk of the debt generating the tax shield and thus the appropriate discount rate for calculating the present value of the tax shield is the risk-free cost of debt.

These three assumptions simplify the calculation of the WACC for the project but may be particularly restrictive for analyzing a project in practice. In some projects, it may be necessary to construct the cashflow statement from the equity point of view in which case it is important to determine the return to equity in each period as a function of the debt-equity ratio for the period. See Tham (1999) for a recent discussion on the formulas for cash flows in perpetuity and for single period cash flows. For a discussion on the adjustment necessary for a project with a finite life and a constant debt-equity ratio, see Ezzell and Miles (1980)

In practical project appraisal, all of these key assumptions may not hold, and thus, the formulas in the textbooks are not applicable and relevant in practice. Clearly, projects in practice do not have infinite lives. For some projects, such as infrastructure projects in power or transport, the life of the project may be 30 years in which case, the assumption of perpetuity may be a suitable approximation. However, in many other projects, the lives of the projects may be less than ten years, in which case it is unclear whether the formulas in perpetuity are suitable approximations for calculating the WACCs for the projects.

The second assumption about the constant debt-equity ratio is more problematic. It may be the case that the debt-equity ratio does vary over the life of the project. It is well known that the required return on equity is a function of the debt-ratio and if the debt-equity ratio does change over the life of the project, then for each period of the project, the return to equity e would have to be calculated. Again, in practice, another assumption is invoked to get around this difficulty by assuming that the change in the debt-equity ratio is small and consequently the change in the value of e is small. And furthermore, even if the change in the debt-ratio is not small, the change in the value of e may be small and thus it is still  not necessary to adjust the formulas and the usual formulas for perpetuity are used. Typically, the textbooks give no examples of the application of the formulas to multiperiod projects.

The multiperiod financial discount rates for a project with a finite length of life is calculated by explicitly taking into account the debt-equity ratio in each period.  In this paper, I will assume that the correct discount rate for the tax shield is $\rho^g$, the required nominal return with all-equity financing. Thus, the financial cashflow can be constructed for a project with any financing scheme and it is **not necessary** to assume that the debt-equity ratio is constant for the life of the project. The format for the construction of the financial statements and the treatment of the impacts of inflation on the cashflow of the project follow the methodology in the manual on cost-benefit analysis by Jenkins and Harberger (1997). It is assumed that the reader is familiar with the methodology. If not, the manual should be consulted for details. The construction of the cashflow statements will be conducted from two points of view: the Total Investment of Point of

View and the Equity Point of View. The abbreviation for the Cashflow Statement from the Total Investment Point of View is CFS-TIP and the abbreviation for the Cashflow Statement from the Equity Point of View is CFS-EPV.

With respect to the calculation of the multiperiod financial discount rates, there are two approaches. We could either assume that the debt equity ratio is constant or variable over the life of the project. If the debt ratio is constant, then over the life of the project, the loan schedule has to be adjusted in such a way that the debt-equity ratio is constant. If we assume that the debt-equity is constant, then the WACC is constant and the return to equity e is also constant.

On the other hand, if the loan schedule for the project is given, then the debt-equity ratio over the life of the project will be a function of the repayment profile for the loan. If the principal is being repaid over the life of the project, then the debt ratio will change over the life of the project. And we know that the nominal return to equity $e^g$ is a function of the debt ratio in each period. Thus, for each period, the nominal return to equity will have to be calculated as a function of the debt ratio for the period. The WACC would have to calculated for each period taking into account the changing debt-equity ratio and the changing value of the return to equity in each period.

In the presence of taxes, we can use the cashflow with or without the tax savings from the interest deduction. In the first method, we use the cashflow with the tax shield and there is no need to adjust the WACC. In the second method, we exclude the tax shield and appropriately adjust the WACC. Both methods give the same valuations. In the following exposition, for simplicity, I will use the first method.

The basic approach for calculating the multiperiod financial discount rates may be summarized as follows:

1. For each period n, estimate $[V^{UL}_{n}]$, the present value of the future cash flows for the unlevered project. Thus, in period n, the present value of the unlevered project is obtained by discounting the future cash flows from period n+1 to period N by $\rho^{g}$, the nominal required return for all-equity financing.

2. For each period n, based on $[V^{UL}_{n}]$ the value of the unlevered project and the desired debt-equity ratio, calculate the amount of debt and equity at the beginning of the period.

3. Based on the amount of debt, construct the loan schedule. From the loan schedule, calculate the annual tax savings from the interest deduction due to the financing.

4. For each period n, estimate $[V^{TS}_{n}]$, the present value of the future tax shields for the unlevered project. Thus, in period n, the present value of the tax shields is obtained by discounting the future tax shields from period n+1 to period N by $\rho^{g}$, the nominal required return for all-equity financing. To obtain $[V^{L}_{n}]$ the value of the levered project in period n, add $[V^{TS}_{n}]$ to $[V^{UL}_{n}]$, the original value of the unlevered project.

5. For each period n, calculate $(e_n^{g})$ the nominal return to equity with financing with the following formula:

$$e_n^{g} = \rho^{g} + (\rho^{g} - d_n^{g})* [D_{n-1}]/[V^{Equity}_{n-1}]$$

6. For each period n, calculate the nominal weighted average cost of capital (WACC) with the following formula:

$$WACC_n^{g} = w_n^{g} = [\%D_{n-1}]*d^{g} + [\%E_{n-1}]*e_n^{g}$$

where $[\%D_{n-1}] = [D_{n-1}]/[V^{L}_{n-1}]$ and $[\%E_{n-1}] = [D_{n-1}]/[V^{L}_{n-1}]$

7. Verify that the NPV of the CFS-TIP (with the tax shield) with $WACC_n^{g}$ is equal to the NPV of the CFS-TIP with $e_n^{g}$.

The paper will be organized as follows. In Section 1, I present the iterative algebraic expressions for the present values of the unlevered project, the present values of the tax shields and the present values of the levered project. In Section 2, using the background from Section 1, I will show that in each period n, the weighted average cost of capital (WACC) is equal to $\rho^g$, the nominal required return with all-equity financing. And for each period n, I derive an expression for $e_n^g$, the nominal return to equity with financing.

**Section 1**

Consider a project with a finite life of N years. The annual real revenues are equal to $R_n$ where n is the nth year of the project. For simplicity assume that the annual operating costs are zero; this assumption does not affect the results. The required investment for the project K is spent at the end of year 0 and there is no reinvestment during the life of the project. The annual tax rate is $\tau$ and the annual tax payments are equal to $T_n$ where n is the nth year of the project. The annual depreciation values are equal to $L_n$ where n is the nth year of the project. The annual inflation rate is g and it is assumed to be constant for the life of the project. For easy reference, the various parameters and assumptions are summarized in Appendix A.

**Financing**

In this case, we are assuming that the debt-equity ratio is constant and thus the loan schedule is constructed in such a way that the debt-equity ratio is constant for the life of the project. However, this is not a critical assumption. For example, if we construct the loan schedule so that the annual repayments on the loan are equal, then the debt-equity ratio will change over the life of the project.

First, we need to determine the value of the unlevered project at any given point in time. The detailed income statement and nominal cashflow statement are given in Appendix B. In period n, the value of the unlevered project is equal to the discounted value of the future cash flows from period n+1 to N. The cash flows are discounted at the nominal required return on equity with all-equity financing $\rho^g$. We will write an iterative expression for the value of the unlevered project in any period n. Since there are N periods for the project, we can write the expressions to calculate the values of the unlevered project for N periods from n = 0 to n = N-1.

**Present Value of the unlevered project in period N-1**

Consider the value of the unlevered project in the N-1 period. The present value of the unlevered project in period N-1 is the discounted value of the cashflow in the Nth period. For the cash flow statement, see Table B2 in Appendix B. Let $\alpha = 1/(1 + \rho^g)$ and $(Z_n) = [(1 - \tau)*(R_n/\gamma^n - L_n) + L_n]$. Then

$$[PV^{UL}_{Yr\ N-1} @ \rho^g] = [V^{UL}_{N-1}] = \text{PV in Year N-1 of CF(Year N)}$$

$$= [(1 - \tau)*(R_N/\gamma^N - L_N) + L_N]*\alpha = (Z_N)*\alpha$$

$$[V^{UL}_{N-1}]*(1 + \rho^g) = (Z_N) \qquad \textbf{(1)}$$

**Present Value of the unlevered project in period N-2**

The present value of the unlevered project in period N-2 is the discounted value of the cashflow in period N-1 and period N.

$[PV^{UL}_{Yr\ N-2} @ \rho^g] = [V^{UL}_{N-2}]$ = PV in Year N-2 of CF(Year N-1 to Year N)

$$= (Z_{N-1})*\alpha + (Z_N)*\alpha^2 \quad \textbf{(2)}$$

Substituting line 1 into line 2, we obtain that

$$[V^{UL}_{N-2}] = (Z_{N-1})*\alpha + [V^{UL}_{N-1}]*\alpha$$

$$[V^{UL}_{N-2}]*(1 + \rho^g) = (Z_{N-1}) + [V^{UL}_{N-1}] \quad \textbf{(3)}$$

Similarly, we can obtain an expression for the present value of the unlevered project in period N-3.

$$[V^{UL}_{N-3}] = (Z_{N-2})*\alpha + (Z_{N-1})*\alpha^2 + (Z_N)*\alpha^3$$

$$= (Z_{N-2})*\alpha + [V^{UL}_{N-2}]*\alpha \quad \textbf{(4)}$$

**Present Value of the unlevered project in period n**

The value of the unlevered project in year n is the discounted value of the cash flows from year n+1 to N. In general, with the exception of period N-1, the value of the unlevered in period n for $0 \leq n \leq N-2$ is given by

$$[V^{UL}_{n}] = (Z_{n+1})*\alpha + [V^{UL}_{n+1}]*\alpha$$

$$[V^{UL}_{n}]*(1 + \rho^g) = (Z_{n+1}) + [V^{UL}_{n+1}] \quad \textbf{(5)}$$

By successively using the iterative formula in line 5, we can show that in year 0, the value of the unlevered project will be the present value of the future cash flows in Year 1 to year N, discounted at $\rho^g$. That is,

$$[V^{UL}{}_{n}] = [(1 - \tau)*(R_1/\gamma - L_1) + L_1]*\alpha + [(1 - \tau)*(R_2/\gamma^2 - L_2) + L_2]*\alpha^2 + .... + [(1 - \tau)*(R_N/\gamma^N - L_N) + L_N]*\alpha^N \quad \textbf{(6)}$$

**Annual amounts of debt for the life of the project**

We will assume that at the beginning of each year of the project, the debt as a percentage of the total value of the unlevered project will be $\chi_n$% and the nominal cost of the debt is $d^g$. Note that in this case, the total value does not include the present value of the tax shield from the financing.

The beginning balance of the debt in year n will be $\chi_n$% of the total value of the unlevered project at the beginning of year n.

**Table 1: Value of debt and equity (beginning of year)**

| | 1 | 2 | . . . . | N-1 | N |
|---|---|---|---|---|---|
| Amount of debt | $D_0$ | $D_1$ | . . . . | $D_{N-2}$ | $D_{N-1}$ |
| Amount of equity | $E_0$ | $E_1$ | . . . . | $E_{N-2}$ | $E_{N-1}$ |
| Total Value (unlevered) | $[V^{UL}{}_{0}]$ | $[V^{UL}{}_{1}]$ | . . . . | $[V^{UL}{}_{N-2}]$ | $[V^{UL}{}_{N-1}]$ |

The values of the debt and equity as a percentage of the unlevered project at different points in time are shown in Table 1.

In any period n, the percentage of debt is

$$\frac{D_{n-1}}{[V^{UL}{}_{n-1}]} = \chi_n\% \quad \textbf{(7)}$$

and the debt-equity ratio is

$$\frac{D_n}{E_n} = \frac{\chi_n\%}{(1 - \chi_n\%)} \quad \textbf{(8)}$$

Based on the debt values in Table 2, we can construct the following loan schedule.

**Table 2: Loan Schedule**

| | 1 | 2 | . . . . | N-1 | N |
|---|---|---|---|---|---|
| Bal, Beg of Year | $B_0$ | $B_1$ | . . . . | $B_{N-2}$ | $B_{N-1}$ |
| Interest Accrued | $d^g*B_0$ | $d^g*B_1$ | . . . . | $d^g*B_{N-2}$ | $d^g*B_{N-1}$ |
| Payment | $P_1$ | $P_2$ | . . . . | $P_{N-1}$ | $P_N$ |
| Bal, End of Year | $B_1$ | $B_2$ | . . . . | $B_{N-1}$ | $B_N$ |

The loan balance in the beginning of year n is equal to the end balance of the year n-1. Also, $D_n$ the amount of debt at the beginning of year n in Table 1 is the same as $B_n$ the beginning balance in year n in the loan schedule in Table 2. The interest accrued in year n is equal to the nominal interest rate times the loan balance in year n-1.

$$d^g*B_{n-1} \quad \textbf{(9)}$$

Thus, in year n, we know that

$$B_{n-1} + d^g*B_{n-1} - P_n = B_n$$

$$(1 + d^g)*B_{n-1} - P_n = B_n \quad \textbf{(10)}$$

And, in general, in year n,

$$B_n = (1 + d^g)*B_{n-1} - P_n \quad \textbf{(11)}$$

Let $\beta = 1/(1 + d^g)$. Thus we can rewrite line 11 as follows

$$P_n = B_{n-1}/\beta - B_n \quad \textbf{(12)}$$

We assume that the loan balance in year N is zero, that is, $B_N = 0$. Thus, in year N, the payment $P_N$ is

$$P_N = B_{N-1}/\beta - B_N = B_{N-1}/\beta \quad \textbf{(13)}$$

With the completion of the loan schedule, we know the annual interest payments that have to be made. The value of the annual tax shield is equal to the tax rate times the annual interest payment. The new tax liabilities, taking into account the annual interest deductions, have to be calculated for each year of the project. We can confirm that the NPV of the financing is zero. See the derivation in Appendix C.

**Total Investment Point of View**

The income statement with the interest deductions from financing is shown in Table B3 in Appendix B. In year n, the tax payment is

$$T_n = \tau*( R_n/\gamma^n - L_n - d^g*B_{n-1})$$

$$= \tau*( R_n/\gamma^n - L_n ) - \tau d^g*B_{n-1} \qquad \textbf{(14)}$$

In year n, the net profits after tax is

$$(R_n/\gamma^n - L_n - d^g*B_{n-1}) - \tau*( R_n/\gamma^n - L_n- d^g*B_{n-1})$$

$$= (1 - \tau)*( R_n/\gamma^n - L_n - d^g*B_{n-1}) \qquad \textbf{(15)}$$

The nominal cashflow statement for the project from the total investment point of view (CFS-TIP) including the tax savings from the interest deduction is shown in Table B4 in Appendix B.

In year n, the net cashflow after tax is

$$R_n/\gamma^n - \tau*( R_n/\gamma^n - L_n - d^g*B_{n-1}) = (1 - \tau)*(R_n/\gamma^n - L_n) + L_n + td^g*B_{n-1}$$

$$= (Z_n) + td^g*B_{n-1} \qquad \textbf{(16)}$$

Compare the expression for the annual net cashflow after tax in line 16 with the net cashflow after tax in line B3. Note that the annual NCF in line 16 is higher than the annual NCF in line B3. The difference is the amount of the annual tax shield which is equal to the tax rate times the annual interest payment.

**Present Value of the tax shield**

To determine the correct value of the levered project, we have to estimate the present value of the tax shield at $\rho^g$ for each year of the project and add these annual values to the annual values of the unlevered project calculated in Table 1. The present values of the tax shield for each period are shown below. Note that the tax shield is discounted at $\rho^g$ and not at the cost of debt $d^g$. See Tham (1999) for a discussion on why it may be appropriate to use $\rho^g$ rather than $d^g$.

**Table 3: Annual Values of the tax shields**

| | 0 | 1 | 2 | . . . . | N |
|---|---|---|---|---|---|
| Tax*interest payments | | $\tau*d^g*B_0$ | $\tau*d^g*B_1$ | . . . . | $\tau*d^g*B_{N-1}$ |

We can write the expressions to calculate the present values of the tax shield for N-1 periods and add these values to the original valuations of the unlevered project given in Table 1.

**Present Value of the tax shield in period N-1**

Consider the N-1 period. The present value of the tax shield (TS) in period N-1 is the discounted value of the tax shield in the Nth period.

$[TS_{Yr\ N-1} @ \rho] = [V^{TS}_{N-1}]$

$= \text{PV in Year N-1 of tax shield(Year N)}$

$= \tau*d^g*B_{N-1}*\alpha$

$$[V^{TS}_{N-1}]*(1 + \rho^g) = \tau*d^g*B_{N-1} \qquad \textbf{(17)}$$

**Present Value of the tax shield in period N-2**

The present value of the unlevered project in period N-2 is the discounted value of the cashflow in period N-1 and period N.

$[TS_{Yr\ N-2} @ \rho] = [V^{TS}_{N-2}]$ = PV in Year N-2 of tax shield (Year N-1 to Year N)

$$= \tau * d^g * B_{N-2} * \alpha + \tau * d^g * B_{N-1} * \alpha^2$$

$$= \tau * d^g * B_{N-2} * \alpha + [V^{TS}_{N-1}] * \alpha$$

$$[V^{TS}_{N-2}] * (1 + \rho^g) = \tau * d^g * B_{N-2} + [V^{TS}_{N-1}] \quad \textbf{(18)}$$

Similarly, we can obtain an expression for the value of the tax shield in period N-3.

$[TS_{Yr\ N-3} @ \rho] = [V^{TS}_{N-3}]$ = PV in Year N-2 of tax shield (Year N-1 to Year N)

$$= \tau * d^g * B_{N-3} * \alpha + \tau * d^g * B_{N-2} * \alpha^2 + \tau * d^g * B_{N-1} * \alpha^3$$

$$= \tau * d^g * B_{N-3} * \alpha + [V^{TS}_{N-2}] * \alpha \quad \textbf{(19)}$$

**Present Value of the tax shield in period n**

And in general, with the exception of period N-1, the value of the tax shield in period n for $0 \leq n \leq N-2$ is given by

$$[V^{TS}_{n}] = \tau * d^g * B_n * \alpha + [V^{TS}_{n+1}] * \alpha$$

$$[V^{TS}_{n}] * (1 + \rho^g) = \tau * d^g * B_n + [V^{TS}_{n+1}] \quad \textbf{(20)}$$

**Table 4: Annual values of the project including the annual values of the tax shield**

| | 0 | 1 | . . . . | N-1 |
|---|---|---|---|---|
| Amount of debt | $D_1$ | $D_2$ | . . . . | $D_{N-1}$ |
| Amount of equity | $E_1$ | $E_2$ | . . . . | $E_{N-1}$ |
| Original Value | $[V^{UL}_{0}]$ | $[V^{UL}_{1}]$ | . . . . | $[V^{UL}_{N-1}]$ |
| PV of Tax shield | $[V^{TS}_{0}]$ | $[V^{TS}_{1}]$ | . . . . | $[V^{TS}_{N-1}]$ |
| Total Value | $[V^{UL}_{0}] + [V^{TS}_{0}]$ | $[V^{UL}_{1}] + [V^{TS}_{1}]$ | | $[V^{UL}_{N-1}] + [V^{TS}_{N-1}]$ |

**Present Value of the levered project in period N-1**

In the N-1 period, the value of the levered project is given by the sum of the cashflow in the Nth period plus the value of the tax shield in the Nth period. For the values of the annual tax shields, see Table 3.

$$[V^L_{N-1}] = \frac{(Z_N) + \tau * d^g * B_{N-1}}{(1 + w_N^g)} \quad \textbf{(21)}$$

where $w_N^g$ is the weighted average cost of capital (WACC) for the Nth period. Line 21 can be rewritten as follows:

$$[V^L_{N-1}]*(1 + w_N^g) = (Z_N) + \tau * d^g * B_{N-1} \quad \textbf{(22)}$$

**Present Value of the levered project in period N-2**

In the N-2 period, the present value of the levered project $[V^L_{N-2}]$ is given by

$$[V^L_{N-2}] = \frac{(Z_{N-1}) + \tau * d^g * B_{N-2}}{(1 + w_{N-1}^g)} + \frac{(Z_N) + \tau * d^g * B_{N-1}}{(1 + w_{N-1}^g)(1 + w_N^g)}$$

$$[V^L_{N-2}]*(1 + w_{N-1}^g) = (Z_{N-1}) + \tau * d^g * B_{N-2} + [V^L_{N-1}] \quad \textbf{(23)}$$

where $w_N^g$ is the weighted average cost of capital (WACC) for period N and $w_{N-1}^g$ is the weighted average cost of capital (WACC) for period N-1.

**Present Value of the levered project in period n**

In general, in period n, the present value of the levered project $[V^L_n]$ is given by

$$[V^L_n] = \frac{(Z_{n+1}) + \tau * d^g * B_n}{(1 + w_{n+1}^g)} + \frac{(V^L_{n+1})}{(1 + w_{n+1}^g)}$$

$$[V^L_n]*(1 + w_{n+1}^g) = (Z_{n+1}) + \tau * d^g * B_n + (V^L_{n+1}) \quad \textbf{(24)}$$

where $w_{n+1}^g$ is the weighted average cost of capital (WACC) for the period n+1.

Previously in calculating the percentage of debt, we did not include the present value of the annual tax shields. If we do include the present value of the annual tax shields, the annual percentage of debt will be lower than the annual percentage debt calculated in Table 1 since $[V^L{}_n] > [V^{UL}{}_n]$ for all values of n.

In any period n, with the tax shield, the percentage of debt would be

$$\frac{D_{n-1}}{[V^L{}_{n-1}]} < \chi_n\% \qquad \textbf{(25)}$$

**Present Values of the Annual Equity Cashflows**

Based on the loan schedule in Table 2, we can construct the cashflow statement from the equity point of view (CFS-EPV).

**Table 5: Nominal Cashflow Statement, Equity Point of View, (CFS-EPV), for the project**

| | 0 | 1 | 2 | . . . . . . . | N |
|---|---|---|---|---|---|
| NCF, TIP | -K | $R_1/\gamma - T_1$ | $R_2/\gamma^2 - T_2$ | . . . . . . . | $R_N/\gamma^N - T_N$ |
| Financing | D | $P_1$ | $P_2$ | . . . . . . . | $P_N$ |
| NCF, Equity | -K+D | $R_1/\gamma - T_1 - P_1$ | $R_2/\gamma^2 - T_2 - P_1$ | . . . . . . | $R_N/\gamma^N - T_N - P_N$ |

Using the results from line 16 and line 12, in year n, for $1 \le n \le N$, the annual net cashflow from the Equity Point of View (CFS-EPV) is given by

$$R_n/\gamma - T_n - P_n = (1 - \tau)*( R_n/\gamma^n - L_n) + L_n + td^g*B_{n-1} - P_n$$

$$= (Z_n) + td^g*B_{n-1} - [B_{n-1}/\beta^g - B_n] \qquad \textbf{(26)}$$

**Present Value of the levered project in period N-1**

Consider the value of the equity in the levered project in the N-1 period. The present value of the equity in the levered project in period N-1 is the discounted value of the cashflow in the Nth period.

$[PV^{Equity}_{Yr\ N-1} @ e_{N-1}] = [V^{Equity}_{N-1}]$ = PV in Year N-1 of CF(Year N)

$$= \frac{(Z_N) + td^g*B_{N-1} - P_N}{(1 + e_N^g)}$$

$$[V^{Equity}_{N-1}]*(1 + e_N^g) = (Z_N) + td^g*B_{N-1} - P_N \qquad \textbf{(27)}$$

**Present Value of the levered project in period N-2**

The present value of the equity in the levered project in period N-2 is the discounted value of the cashflow in the N-1 period to the Nth period.

$[PV^{Equity}_{Yr\ N-2} @ e] = [V^{Equity}_{N-2}]$ = PV in Year N-2 of CF(Year N-1 to N)

$$= \frac{(Z_{N-1}) + td^g*B_{N-2} - P_{N-1}}{(1 + e_{N-1}^g)} + \frac{(Z_N) + td^g*B_{N-1} - P_N}{(1 + e_{N-1}^g)*(1 + e_N^g)}$$

$$[V^{Equity}_{N-2}]*(1 + e_{N-1}^g) = (Z_{N-1}) + td^g*B_{N-2} - P_{N-1} + [V^{Equity}_{N-1}] \qquad \textbf{(28)}$$

In general, we can show that for any period n,

$$[V^{Equity}_{n}]*(1 + e_{n+1}^g) = (Z_{n+1}) + td^g*B_n - P_{n+1} + [V^{Equity}_{n+1}] \qquad \textbf{(29)}$$

**Section 2**

In this section, using the algebraic expressions from Section 1, I show that in each period n, the weighted average cost of capital (WACC) is equal to $\rho^g$, the nominal required return with all-equity financing. And for each period n, I derive an expression for $e_n^g$, the nominal return to equity with financing. For each period n, we will assume that the present value of the levered project is equal to the present value of the unlevered project plus the present value of the tax shield.

**Period N-1**

In the N-1 period, the value of the levered project is given by

$[V^{L}_{N-1}] = [V^{UL}_{N-1}] + [V^{TS}_{N-1}]$

$$[V^{L}_{N-1}] = [V^{UL}_{N-1}] + \frac{\tau * d^{g} * B_{N-1}}{(1 + \rho^{g})} \quad \textbf{(30)}$$

We can rewrite line 30 as follows:

$$[V^{L}_{N-1}]*(1 + \rho^{g}) = [V^{UL}_{N-1}]*(1 + \rho^{g}) + \tau * d^{g} * B_{N-1} \quad \textbf{(31)}$$

Substituting line 1 into line 31, we obtain:

$$[V^{L}_{N-1}]*(1 + \rho^{g}) = (Z_{N}) + \tau * d^{g} * B_{N-1} \quad \textbf{(32)}$$

Comparing line 22 and line 32, we can conclude that $w_{N}^{g}$ is equal to $\rho^{g}$.

**Period N-2**

In the N-2 period, the value of the levered project is given by

$$[V^{L}_{N-2}] = [V^{UL}_{N-2}] + [V^{TS}_{N-2}] \quad \textbf{(33)}$$

Using the expression for the present value of the unlevered project in line 3 and the expression for the present value of the tax shield in line 18, we can rewrite line 33 as follows

$[V^{L}_{N-2}] = (Z_{N-1})*\alpha + [V^{UL}_{N-1}]*\alpha + \tau * d^{g} * B_{N-2}*\alpha + [V^{TS}_{N-1}]*\alpha$

$$[V^{L}_{N-2}]*(1 + \rho^{g}) = (Z_{N-1}) + \tau * d^{g} * B_{N-2} + [V^{UL}_{N-1}] + [V^{TS}_{N-1}] \quad \textbf{(34)}$$

Using line 30, we can rewrite line 34 as follows

$$[V^{L}_{N-2}]*(1 + \rho^{g}) = (Z_{N-1}) + \tau * d^{g} * B_{N-2} + [V^{L}_{N-1}] \quad \textbf{(35)}$$

Comparing line 23 and line 35, we can conclude that $w_{N-1}^{g}$ is equal to $\rho^{g}$.

**Period n**

In general, in period n, the value of the levered project is given by

$[V^{L}{}_{n}] = [V^{UL}{}_{n}] + [V^{TS}{}_{n}]$ **(36)**

Using line 5 and line 20, we can rewrite line 36 as follows

$[V^{L}{}_{n}] = (Z_n)*\alpha + [V^{UL}{}_{n+1}]*\alpha + \tau*d^{g}*B_N*\alpha + [V^{TS}{}_{N+1}]*\alpha$

$[V^{L}{}_{n}]*(1 + \rho^{g}) = (Z_n) + \tau*d^{g}*B_n + [V^{UL}{}_{n+1}] + [V^{TS}{}_{n+1}]$

$[V^{L}{}_{n}]*(1 + \rho^{g}) = (Z_n) + \tau*d^{g}*B_n + [V^{L}{}_{n+1}]$ **(37)**

Comparing line 24 and line 37, we can conclude that $w_{n+1}{}^{g}$ is equal to $\rho^{g}$. And this is true for all periods.

**Nominal return to equity with financing $e_n{}^{g}$**

Next, I will write an expression for $e_n{}^{g}$, the nominal return to equity with financing. For the N-1 period, we can write two expressions for the present value of the cash flows of the levered project.

$[V^{L}{}_{N-1}] = [V^{UL}{}_{N-1}] + [V^{TS}{}_{N-1}]$ **(38)**

$[V^{L}{}_{N-1}] = [V^{Equity}{}_{N-1}] + [V^{Debt}{}_{N-1}]$ **(39)**

Combining line 38 and line 39, we obtain that

$[V^{Equity}{}_{N-1}] + [V^{Debt}{}_{N-1}] = [V^{UL}{}_{N-1}] + [V^{TS}{}_{N-1}]$ **(40)**

We can rewrite line 40 as follows:

$[V^{Equity}{}_{N-1}] = [V^{UL}{}_{N-1}] + [V^{TS}{}_{N-1}] - [V^{Debt}{}_{N-1}]$ **(41)**

For convenience, we will rewrite the expressions for each of the terms in line 41.

$[V^{UL}{}_{N-1}]*(1 + \rho^{g}) = (Z_N)$ **(42)**

$[V^{TS}{}_{N-1}]*(1 + \rho^{g}) = \tau*d^{g}*B_{N-1}$ **(43)**

$$[V^{Equity}{}_{N-1}]*(1 + e_N{}^g) = (Z_N) + td^g*B_{N-1} - P_N \quad \textbf{(44)}$$

$$[V^{Debt}{}_{N-1}]*(1 + d^g) = P_N \quad \textbf{(45)}$$

See line 1, line 17, line 27 and line C4. Substitute line 42, line 43 and line 45 into line 44 and simplify to obtain.

$$[V^{Equity}{}_{N-1}]*(1 + e_N{}^g) = (Z_N) + td^g*B_{N-1} - P_N$$

$$= [V^{UL}{}_{N-1}]*(1 + \rho^g) + [V^{TS}{}_{N-1}]*(1 + \rho^g) - [V^{Debt}{}_{N-1}]*(1 + d^g) \quad \textbf{(46)}$$

Subtract line 37 from line 46 and simplify to obtain

$$[V^{Equity}{}_{N-1}]*e_N{}^g = [V^{UL}{}_{N-1}]*\rho^g + [V^{TS}{}_{N-1}]*\rho^g - [V^{Debt}{}_{N-1}]*d^g$$

$$[V^{Equity}{}_{N-1}]*e_N{}^g = \{[V^{UL}{}_{N-1}] + [V^{TS}{}_{N-1}]\}*\rho^g - [V^{Debt}{}_{N-1}]*d^g \quad \textbf{(47)}$$

Substituting line 37 into line 47 we obtain

$$[V^{Equity}{}_{N-1}]*e_N{}^g = [V^{Equity}{}_{N-1}]*\rho^g + [V^{Debt}{}_{N-1}]*\rho^g - [V^{Debt}{}_{N-1}]*d^g \quad \textbf{(48)}$$

Solving for $e_N{}^g$, we obtain

$$e_N{}^g = \rho^g + (\rho^g - d^g)*\frac{[V^{Debt}{}_{N-1}]}{[V^{Equity}{}_{N-1}]} \quad \textbf{(49)}$$

**Period n < N-1**

For any period n < N-1, we can follow a similar process and write two expressions for the present value of the cash flows of the levered project.

$$[V^L{}_n] = [V^{UL}{}_n] + [V^{TS}{}_n] \quad \textbf{(50)}$$

$$[V^L{}_n] = [V^{Equity}{}_n] + [V^{Debt}{}_n] \quad \textbf{(51)}$$

Combining line 50 and line 51, we obtain that

$$[V^{Equity}{}_{n}] + [V^{Debt}{}_{n}] = [V^{UL}{}_{n}] + [V^{TS}{}_{n}] \quad \textbf{(52)}$$

We can rewrite line 52 as follows:

$$[V^{Equity}{}_{n}] = [V^{UL}{}_{n}] + [V^{TS}{}_{n}] - [V^{Debt}{}_{n}] \quad \textbf{(53)}$$

For convenience, we will rewrite the expressions for each of the terms in line 53.

$$[V^{UL}{}_{n}]*(1 + \rho^{g}) = (Z_{n+1}) + [V^{UL}{}_{n+1}] \quad \textbf{(54)}$$

$$[V^{TS}{}_{n}]*(1 + \rho^{g}) = \tau*d^{g}*B_{n} + [V^{TS}{}_{n+1}] \quad \textbf{(55)}$$

$$[V^{Equity}{}_{n}]*(1 + e_{n+1}{}^{g}) = (Z_{n+1}) + td^{g}*B_{n} - P_{n+1} + [V^{Equity}{}_{n+1}] \quad \textbf{(56)}$$

$$[V^{Debt}{}_{n}]*(1 + d^{g}) = P_{n+1} + [V^{Debt}{}_{n+1}] \quad \textbf{(57)}$$

See line 5, line 20, line 28 and line C6. Substitute line 54, line 55 and line 57 into line 56 and simplify to obtain.

$$[V^{Equity}{}_{n}]*(1 + e_{n+1}{}^{g}) = (Z_{n+1}) + td^{g}*B_{n} - P_{n+1} + [V^{Equity}{}_{n+1}]$$

$$= [V^{UL}{}_{n}]*(1 + \rho^{g}) - [V^{UL}{}_{n+1}] + [V^{TS}{}_{n}]*(1 + \rho^{g}) - [V^{TS}{}_{n+1}]$$

$$- [V^{Debt}{}_{n}]*(1 + d^{g}) + [V^{Debt}{}_{n+1}] \quad \textbf{(58)}$$

Rewriting line 58, we obtain

$$[V^{Equity}{}_{n}]*(1 + e_{n+1}{}^{g}) = [V^{UL}{}_{n}]*(1 + \rho^{g}) + [V^{TS}{}_{n}]*(1 + \rho^{g}) - [V^{Debt}{}_{n}]*(1 + d^{g})$$

$$+ \{[V^{Equity}{}_{n+1}] - [V^{UL}{}_{n+1}] - [V^{TS}{}_{n+1}] + [V^{Debt}{}_{n+1}]\} \quad \textbf{(59)}$$

For n = N-2, the expression in curly brackets on the right side of line 59 is equal to zero. We can subtract line 50 from line 59 and simplify to obtain

$$[V^{Equity}{}_{n}]*e_{n+1}{}^{g} = [V^{UL}{}_{n}]*\rho^{g} + [V^{TS}{}_{n}]*\rho^{g} - [V^{Debt}{}_{n}]*d^{g}$$

$$[V^{Equity}{}_{n}]*e_{n+1}{}^{g} = \{[V^{UL}{}_{n}] + [V^{TS}{}_{n}]\}*\rho^{g} - [V^{Debt}{}_{n}]*d^{g} \quad \textbf{(60)}$$

Substituting line 37 into line 60 we obtain

$$[V^{Equity}{}_{n}]*e_{n+1}{}^{g} = [V^{Equity}{}_{n}]*\rho^{g} + [V^{Debt}{}_{n}]*\rho^{g} - [V^{Debt}{}_{n}]*d^{g} \quad \textbf{(61)}$$

Solving for $e_{n+1}{}^{g}$, we obtain

$$e_{n+1}{}^{g} = \rho^{g} + (\rho^{g} - d^{g})*\frac{[V^{Debt}{}_{n}]}{[V^{Equity}{}_{n}]} \quad \textbf{(62)}$$

Continuing in this iterative fashion, we can show that the expression for the return to equity in line 62 holds for any period.

## CONCLUSION

In conclusion, we have shown that, based on any financing profile, the annual returns to equity with financing can be explicitly calculated for any multiperiod project. In this way, the cash flow statement from the Equity Point of View (CFS-EPV) can be constructed.

## Appendix A

## SUMMARY OF THE BASIC PARAMETERS AND ASSUMPTIONS

### Initial Investment Cost and Annual Depreciation

The cost of the investment at the end of year 0 is K.
The life of the project is N years.
The economic life of the investment in machinery is N years and thus there is no liquidation value at the end of the project. The annual depreciation is $L_t$. If we assume straight line depreciation, then $L = L_i = L_j = K/N$ for $i, j \le N$. But it is not necessary to assume straight line depreciation.

### Annual revenues

In period n, the annual **real** revenue is $R_n$ and is expected to rise with the rate of inflation. For simplicity, there are no Accounts Receivable, no Accounts Payable and no cash balance; in addition, the annual operating costs are zero. If the annual revenues are constant, then $R = R_i = R_j$ for $i, j \le N$. where N is the number of periods for the project.

### Tax rate and inflation rate

The annual tax rate is $\tau_n$.
The annual inflation rate is g and $\gamma = 1/(1+g)$. For simplicity, we assume that the annual tax rate and annual inflation rate are constant for the life of the project.

### Return on Equity

The required nominal return on all-equity financing is $\rho^g = \rho^r*(1+g) + g$. Let $\alpha = 1/(1 + \rho^g)$.
The required real return on all-equity financing is $\rho^r$ and is constant for the life of the project. And $(1 + \rho^g) = (1 + \rho^r)*(1+g)$ .
The real return to equity (with financing) in period n is $e_n^r$.
The nominal return to equity (with financing) in period n is $e_n^g = e_n^r*(1+g) + g$.

### Cost of Debt

The real cost of debt financing is $d^r$ and $\beta = 1/(1+d^g)$.
The nominal cost of debt is $d^g = d^r*(1+g) + g$.

**Appendix B**

The income statement for the project with no financing is shown in Table B1.

**Table B1: Income Statement for the project with no interest deduction**

| | 0 | 1 | 2 | . . . . | N |
|---|---|---|---|---|---|
| Revenues | | $R_1/\gamma$ | $R_2/\gamma^2$ | . . . . | $R_N/\gamma^N$ |
| Depreciation | | $L_1$ | $L_2$ | . . . . | $L_N$ |
| Interest deduction | | 0 | 0 | | 0 |
| Net Profits before tax | | $R_1/\gamma - L_1$ | $R_2/\gamma^2 - L_2$ | . . . . | $R_N/\gamma^N - L_N$ |
| Tax | | $T_1$ | $T_2$ | . . . . | $T_N$ |
| Net Profits after tax | | $(R_1/\gamma - L_1) - T_1$ | $(R_2/\gamma^2 - L_2) - T_2$ | . . . . | $(R_N/\gamma^N - L_N) - T_N$ |

In year n, the tax payment, with no interest deduction, is

$$T_n = \tau^*( R_n/\gamma^n - L_n) \qquad \textbf{(B1)}$$

In year n, the net profits after tax is

$$(R_n/\gamma^n - L_n) - T_n = (R_n/\gamma^n - L_n) - \tau^*( R_n/\gamma^n - L_n)$$

$$= (1 - \tau)^*( R_n/\gamma^n - L_n) \qquad \textbf{(B2)}$$

**Table B2: Nominal cashflow Statement for the project with no financing**

| | 0 | 1 | 2 | . . . . . . . | N |
|---|---|---|---|---|---|
| Revenues | | $R_1/\gamma$ | $R_2/\gamma^2$ | . . . . . . . | $R_N/\gamma^N$ |
| Investment | K | | | | |
| NCF before tax | -K | $R_1/\gamma$ | $R_2/\gamma^2$ | . . . . . . . | $R_N/\gamma^N$ |
| Tax | | $T_1$ | $T_2$ | . . . . . . . | $T_N$ |
| NCF | -K | $R_1/\gamma - T_1$ | $R_2/\gamma^2 - T_2$ | . . . . . . . | $R_N/\gamma^N - T_N$ |

The cashflow statement for the project from the Total Investment Point of view (CFS-TIP) with no financing is shown in Table B2. This is the same as the cashflow statement from the All-Equity Point of View (CFS-AEPV).

In year n, the annual net cashflow after tax is

$$R_n/\gamma^n - T_n = R_n/\gamma^n - \tau^*( R_n/\gamma^n - L_n)$$

$$= (1 - \tau)^*(R_n/\gamma^n - L_n) + L_n \qquad \textbf{(B3)}$$

Compare the expression for the net profits after tax in line B2 with the net cashflow after tax in line B3. The difference is the amount of the annual depreciation $L_n$.

If there is no financing, then in year 0, the NPV of the unlevered project at $\rho^g$, the nominal required return on all-equity financing is equal to the discounted values of the cash flows from year 1 to year N.

$$NPV^{UL}_{Yr0} \text{ @ } \rho = -K + \frac{[(1 - \tau)*(R_1/\gamma - L_1) + L_1]}{(1 + \rho^g)}$$

$$+ \frac{[(1 - \tau)*(R_2/\gamma^2 - L_2) + L_2]}{(1 + \rho^g)^2} + .... + \frac{[(1 - \tau)*(R_N/\gamma^N - L_N) + L_N]}{(1 + \rho^g)^N} \qquad \textbf{(B4)}$$

Let $(Z_n) = [(1 - \tau)*(R_n/\gamma^n - L_n) + L_n]$ and $\alpha = 1/(1 + \rho^g)$. Then

$$NPV^{UL}_{Yr0} \text{ @ } \rho = -K + (Z_1)*\alpha + (Z_2)*\alpha^2 + .... +$$

$$+ (Z_{N-1})*\alpha^{N-1} + (Z_N)*\alpha^N \qquad \textbf{(B5)}$$

The income statement with the interest deductions from financing is shown in Table B3.

**Table B3: Income Statement for the project taking into account the interest deductions from financing**

| | 0 | 1 | 2 | . . . . | N |
|---|---|---|---|---|---|
| Revenues | | $R_1/\gamma$ | $R_2/\gamma^2$ | . . . . | $R_N/\gamma^N$ |
| Depreciation | | $L_1$ | $L_2$ | . . . . | $L_N$ |
| Interest deduction | | $d^g*B_0$ | $d^g*B_1$ | . . . . | $d^g*B_{N-1}$ |
| Net Profits before tax | | $R_1/\gamma - L_1 - d^g*B_0$ | $R_2/\gamma^2 - L_2 - d^g*B_1$ | | $R_N/\gamma^N - L_N - d^g*B_{N-1}$ |
| Tax | | $T_1$ | $T_2$ | . . . . | $T_N$ |
| Net Profits after tax | | $(R_1/\gamma - L_1 - d^g*B_0) - T_1$ | $(R_2/\gamma^2 - L_2 - d^g*B_1) - T_2$ | | $(R_N/\gamma^N - L_N - d^g*B_{N-1}) - T_N$ |

The nominal cashflow statement for the project from the total investment point of view (CFS-TIP) including the tax savings from the interest deduction is shown in Table B4.

| Table B4: Nominal Cashflow Statement, Total Investment Point of View, (CFS-TIP) | | | | | |
|---|---|---|---|---|---|
| | 0 | 1 | 2 | . . . . . . . | N |
| Revenues | | $R_1/\gamma$ | $R_2/\gamma^2$ | . . . . . . . | $R_N/\gamma^N$ |
| Investment | K | | | | |
| NCF before tax | -K | $R_1/\gamma$ | $R_2/\gamma^2$ | . . . . . . . | $R_N/\gamma^N$ |
| Tax | | $T_1$ | $T_2$ | . . . . . . . | $T_N$ |
| NCF | -K | $R_1/\gamma - T_1$ | $R_2/\gamma^2 - T_2$ | . . . . . . . | $R_N/\gamma^N - T_N$ |

Compare the tax liabilities in Table B3 with the tax liabilities in Table B1 in Appendix B. The difference in the annual tax payments is the value of the annual tax shield. In each year of the project, the tax payment is lower due to the tax shield from the interest deduction. Thus, the present value of the tax payments in Table B3 will be lower than the present value of the tax payments in Table B1.

**Appendix C**

We can confirm that the NPV of the financing is zero. Let $\beta = 1/(1 + d^g)$.

$$[NPV^{Debt}{}_{Yr\,0} @ d^g] = B_0 - \frac{P_1}{(1 + d^g)} - \frac{P_2}{(1 + d^g)^2} \;....\; - \frac{P_{N-1}}{(1 + d^g)^{N-1}} - \frac{P_N}{(1 + d^g)^N}$$

$$[NPV^{Debt}{}_{Yr\,0} @ d^g] = B_0 - P_1*\beta - P_2*\beta^2 \;.....\; - P_{N-1}*\beta^{N-1} - P_N*\beta^N \qquad \textbf{(C1)}$$

Using the expression for the payment in line 12, we obtain

$$[NPV^{Debt}{}_{Yr\,0} @ d^g] = B_0 - [B_0/\beta - B_1]*\beta - [B_1/\beta - B_2]*\beta^2 \;.....$$

$$- [B_{N-2}/\beta - B_{N-1}]*\beta^{N-1} - [B_{N-1}/\beta - B_N]*\beta^N \qquad \textbf{(C2)}$$

By simplifying the parentheses and collecting terms, we see that

$$[NPV^{Debt}{}_{Yr\,0} @ d^g] = B_N*\beta^N = 0 \qquad \textbf{(C3)}$$

Consider the value of the debt in the N-1 period. The present value of the debt in period N-1 is the discounted value of the repayment in the Nth period.

$$[V^{Debt}{}_{N-1}] = P_N*\beta$$

$$[V^{Debt}{}_{N-1}]*(1 + d^g) = P_N \qquad \textbf{(C4)}$$

In the N-2 period, the value of the debt is equal to the discounted value of the repayment in period N-1 and period N.

$$[V^{Debt}{}_{N-2}] = P_{N-1}*\beta + P_N*\beta^2$$

$$= P_{N-1}*\beta + [V^{Debt}{}_{N-1}]*\beta^2 \qquad \textbf{(C5)}$$

And in general, in period n, for $0 \le n \le N-1$ the value of the debt is

$$[V^{Debt}{}_{n}] = P_{n+1}*\beta + P_{n+2}*\beta^2 + \;......\; + P_{N-1}*\beta^{N-(n+1)} + P_N*\beta^{N-n}$$

$$= P_n*\beta + [V^{Debt}{}_{n+1}]*\beta \qquad \textbf{(C6)}$$

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