

# Development Discussion Papers

## **Return to Equity in Project Finance for Infrastructure**

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## INTRODUCTION

In recent years, all over the world, the role of the private sector in the financing of projects in the infrastructure sector has increased tremendously.<sup>1</sup> The financial crisis has slowed the investment flow into infrastructure, but with economic recovery, it is expected that the demand for infrastructure services will be high and the stream of private investments into infrastructure will resume.

In this paper, I present a practical framework for estimating the cost of capital in project finance without the usual simplifying assumptions.<sup>2</sup> The current approach extends the analysis of projects and the construction of cash flow statements in several important respects.

First, I will assume a project with a finite life. Thus, we can directly model the annual cash flows for a infrastructure project with any length of life. Typically, it is assumed that the project has an infinite life with constant annual cash flows. But this analytic simplification deprives us of the ability to model and analyze the annual profile of the cash flows of the project from the point of view of the specific stakeholders in the project.<sup>3</sup>

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<sup>1</sup> For a recent report on infrastructure in Asia, see the publication by the Australian Agency for International Development (1998).

<sup>2</sup> Examples of the typical analysis with the usual simplifications are given in Klein (pg 7, 1997).

<sup>3</sup> In infrastructure structure, there may be wide annual fluctuation in the cash flows over the life of the project. For example, the annual cash flows in a road project is a function of the annual demand and the tariff structure over the life of the project. In many developing countries, the domestic inflation rate is volatile and the tariff structure may be heavily regulated. Thus, it would be a gross oversimplification to assume constant real annual cash flows in the analysis, even if the "basic arguments" are unchanged. See Klein (pg 7, 1997) The simplified analysis is potentially misleading and does not provide even a close approximation to the likely nature of the cash flows from the project.

Second, I will assume that the investments in construction occur over many periods rather than at a point of time in a single period. This means that the annual values of equity and debt will have to be determined during the period of construction.<sup>4</sup>

Third, I will assume that a certain percentage of the annual investment costs will be financed by debt and the debt-equity ratio varies over the life of the project. In the typical analysis, the debt-equity ratio is held constant for the life of the project. Again, this analytic simplification of a constant debt-equity ratio implies that the return to equity is constant and allows us to calculate easily the weighted average cost of capital.

However, it is more realistic to assume that the financing arrangements are based on the investment costs rather than on the value of the expected future cash flows and thus, the debt-equity ratio will not be constant over the life of the project. Moreover, the repayment of the debt over the life of the project will affect the annual debt-equity ratios. By directly estimating the annual debt-equity ratios, we can explicitly examine the impact of the financing on the annual outcomes of the project and the required returns to equity. Based on the annual values of the unlevered cash flows and the specific borrowing profile of the project financing, we can determine the annual values of the equity and the annual required returns to equity, as a function of the varying annual debt-equity ratios.<sup>5</sup>

Fourth, in line with Miller and Modigliani's Theorem, I will assume that the valuation of the project is unaffected by financing, that is, the annual value of the unlevered project is equal to the annual value of the levered project. This assumption will

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<sup>4</sup> If all of the construction occurs in one point in time in a single year, the annual valuations of the project are simply the discounted values of the future cash flows. However, in the case where the investments are spread over many years, the estimation of the values of the project during the early years of construction have to take into account the investments that have to be made during the later years of construction.

<sup>5</sup> It is important to note that with financing the risk analysis will involve censored probability distributions for the lender to the project and the equity holder. In any year, if the cash flow is less than the annual loan repayments, the cash flow to the equity holder is zero; in any year, if the cash flow is more than the annual loan repayments, the cash flow to the equity holder is equal to the cash flow less the loan repayment. For the equity holder, the probability distribution of the cash flow is censored from below. For the lender, if the cash flow is greater than the loan repayment, the lender receives the loan repayment; if the cash flow is less than the loan repayment, the lender receives the cash flow. For the lender, the probability distribution of the cash flow is censored from above.

allow us to establish a relationship between the return to equity for an unlevered project and the return to equity for a levered project. In developing countries without well-functioning capital markets the stringent assumptions of the M & M framework may not be appropriate, but the use of the M & M framework is only the starting point of the analysis. From this starting point, we can relax the assumptions step by step and examine how the outcomes and viability of the project are affected.

Finally, the construction of the annual cash flows and the estimation of the annual returns from the point of view of the various stakeholders will provide better opportunities to model and examine the impact of the risk profile of the project for the different stakeholder under different scenarios.

Since the focus of the paper is on the cost of capital in the context of project finance, I will assume that there are no taxes, no inflation, and no foreign exchange risks. With minor modifications, these additional issues can be incorporated easily into the current analysis.

One of the distinguishing feature of the investment in infrastructure is the use of project finance, that is, the projects are financed based on the projected future cash flows from the project rather than on the basis of the company's balance. For a recent discussion on project finance, see the report by the International Finance Corporation (IFC, 1999).<sup>7</sup>

In project finance, the construction of the annual cash flow and the estimation of the annual returns to the equity are particularly important because the risk analysis of the project for the equity holder requires the estimation of the actual annual cash flow that will accrue to the equity holder. Often, there are guarantees and other government actions

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<sup>6</sup> Furthermore, this assumption implies that the net present value of the unlevered cash flow, discounted at the unlevered cost of capital will always be equal to the net present value of the equity cash flows, discounted at the appropriate annual returns to equity.

<sup>7</sup> If the project is financed on the basis of the company's balance sheet, the construction of the cash flow and the estimation of the cost of capital for a specific project is not relevant; the appropriate cost of capital is the cost of capital for the company, and with the assumption of a constant debt-equity ratio for the company, the return to the equity will also be constant.

and these actions affect the annual cash flows and thus, it is necessary to calculate the annual returns to equity. It is not sufficient to assume that the return to equity is constant; the return to equity is a function of the debt-equity ratio and the debt-equity ratio varies from year to year<sup>8</sup>.

In this framework, I will introduce a table called the free cash flow (FCF) schedule. This is also known as the Total Investment Point of View Schedule or TIP schedule<sup>9</sup>. The TIP (or FCF) schedule is similar in structure to a loan schedule and explicitly shows the annual values of the unlevered project. With the help of the TIP and loan schedules, we can determine the annual values of the equity, and consequently estimate the annual returns to equity. This is a crucial difference between the typical analysis and the current approach.

In Section One, I present two equivalent ways of calculating the annual values of the future unlevered cash flows of the project. First, it will be based on the cash flow approach, and second, based on the TIP schedule. Then I will introduce the financing profile and consequently determine the value of the equity<sup>10</sup>.

In Section Two, I will discuss some practical applications of this approach discussed in Section One. Consider the determination of a fair tariff for a BOT (Build Operate Transfer) project in power. In actual practice, it may be very difficult to determine the exact financing profile over the life of the project. And since the calculation of the required annual returns to equity depends on knowing the annual debt-equity ratios over the life of the project, it may not be possible to calculate the annual returns to equity. As an approximation, the regulators may be willing to negotiate a constant return to

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<sup>8</sup> As an alternative, one may use the IRR of the equity cash flow but the IRR of the equity cash flow has no easy interpretation. What is the meaning of the IRR for the equity cash flow? It is a summary statistic but it does not properly take into account the annual changes in the value of equity and thus the annual returns to equity.

<sup>9</sup> For a theoretical and general discussion of the cost of capital, see Tham (1999a).

<sup>10</sup> For ease of exposition in Section One, I will assume that the NPV of the project is zero. In Appendix A, I will briefly outline the general analytic approach for a project with any NPV. One of the strengths of the TIP schedule is the availability of the annual values of the equity contribution. There are two ways to estimate the annual values of the future cash flows.

equity that does not depend on the varying annual debt-equity ratio. But what debt-equity ratio should be used to estimate the return to equity? If the initial debt-equity ratio is used, and in fact over the life of the project the debt-equity ratio declines, then the return to the equity holder, based on the initial debt-equity ratio would be overstated. With the approach in this paper, we can determine the constant return to equity that is equivalent in value to the varying annual returns to equity.

**SECTION ONE**

Consider an infrastructure project that requires investments at the end of year 0, year 1 and year 2, and generates cash flows in year 3 through year 5. The constant annual revenues was chosen to provide a return of 10 percent. In other words, the net present value (NPV) of the free cash flow is zero. The free cash flow (also known as the cash flow from the Total Investment Point of View) is the unlevered cash flow and discounted at  $\rho$ , the required return to equity. In many infrastructure projects, the negotiation of the tariff structure over the life of the project is based on the assumption about the cost of capital. Thus, if we assume that the relevant cost of capital is ten percent, then based on this, we can estimate the annual revenues that would required to provide a return of 10% (or an NPV of zero).<sup>11</sup>

**Table 1: Free Cash Flow (FCF) Statement or TIP Statement**

	Year>	0	1	2	3	4	5
Annual Revenues					830.37	830.37	830.37
Annual Investments		500.00	600.00	800.00			
Net Cash Flow		-500.00	-600.00	-800.00	830.37	830.37	830.37
NPV @ 10.0 %, yr		0.00					

The cash flow statement is shown above. One of the weakness of the cash flow statement is that it does not show the annual values of the future cash flows and in order to calculate the annual returns to equity, we have to estimate the annual present values of the levered future cash flows from the point of view of the equity holder. Later, the construction of the TIP schedule will permit the estimation of the annual values of equity.

In the usual case, the value of the cash flow at the end of year  $t$  is the present value of future cash flows from year  $t+1$  to  $N$ , the last year of the project. For example, at the end of year three (or the beginning of year four), the value of the unlevered project would be the present value of the cash flows in year four and year five.

$$V^{UL}(\text{end year 3, of future cash flows}) = \frac{R_4}{(1 + \rho)} + \frac{R_5}{(1 + \rho)^2} \quad (1)$$

where  $V^{UL}$  is the valued of the unlevered future cash flows,  $\rho$  is the required return for unlevered cash flows, and  $R_4$  and  $R_5$  are the revenues in year four and year five respectively.

And at the end of year two, the value of the unlevered project would be the present value of the cash flows in year three through year five.

$$\begin{aligned} V^{UL}(\text{end of year 2}) &= \frac{R_3}{(1 + \rho)} + \frac{R_4}{(1 + \rho)^2} + \frac{R_5}{(1 + \rho)^2} \\ &= \frac{R_3 + V^{UL}(\text{in year 3})}{(1 + \rho)} \end{aligned} \quad (2)$$

What is the value of the unlevered project in year one? The natural inclination is simply to discount the future cash flows in year three through year five to year one.

$$\begin{aligned} V^{UL}(\text{end of year 1}) &= \frac{R_3}{(1 + \rho)^2} + \frac{R_4}{(1 + \rho)^3} + \frac{R_5}{(1 + \rho)^4} \\ &= \frac{R_3 + V^{UL}(\text{in year 3})}{(1 + \rho)^2} \end{aligned} \quad (3)$$

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<sup>11</sup>. In the Appendix, we will relax this assumption and demonstrate that this approach works for a project with any NPV and with any revenue profile, and it is not necessary to assume that the annual revenues are constant.

Based on the above numerical example, the annual present values of the future cash flows with respect to a given year t are shown below in Table 2. At the end of year two, the present value of the future cash flows is equal to \$2,065. What is the meaning of this value? This is the amount of money that we would be willing to pay at the end of year two in order to receive the cash flows in year three through year five. Extending the same idea to year one and year zero, we might be **tempted** to say that at the end of year 1, we would be willing to pay \$1,877.3, and at the end of year zero, we would be willing to pay \$1,706.6. However, note that at the end of year zero, the amount of money invested is only \$500 whereas at the end of year 0, the present value of the future cash flows from year three through year five is more than three times the value of the investment.

**Table 2a: Present Values in year t of future cash flows from year t+1 until the end of the project**

	Year>	0	1	2	3	4	5
Annual Revenues			0.00	0.00	830.37	830.37	830.37
PV in year t		1,706.6	1,877.3	2,065.0	1,441.1	754.9	

To solve this apparent puzzle, we have to recognize that investments are made at the end of year one and year two, and the value of the unlevered project at the end of year zero has to take that into account. Thus, the correct value of the unlevered project in year one must include the value of the investment at the end of year two, discounted to year one.

In the typical analysis, this issue does not arise because all of the investments are made in a single year and does not occur over several years. However, in cases where the investment costs are spread over many years, the calculation of the present values must recognize the investments required. Thus, at the end of year one, the correct expression for calculating the present value of the unlevered future cash flows is as follows:

$$V^{UL}(\text{in year 1}) = \frac{-K_3}{(1 + \rho)} + \frac{R_3}{(1 + \rho)^2} + \frac{R_4}{(1 + \rho)^3} + \frac{R_5}{(1 + \rho)^4} \quad (4)$$

The correct values of the unlevered cash flow at time  $t$ , taking into account the investments made over many periods, is shown in Table 2b below.

**Table 2b: Present Values in year  $t$  of future cash flows from year  $t+1$  until the end of the project**

	Year>	0	1	2	3	4	5
Annual Cash Flows			-600.00	-800.00	830.37	830.37	830.37
PV in year $t$		500.0	1,150.0	2,065.0	1,441.1	754.9	

Another equivalent way to think about the annual values of the unlevered project is to construct a free cash flow (FCF) or TIP schedule. The TIP schedule is shown below.

**Table 3: The TIP Schedule**

	Year>	0	1	2	3	4	5
Annual Investments		500.00	600.00	800.00			
Beg Balance			500.00	1,150.00	2,065.00	1,441.13	754.88
Dividend Accrued			50.00	115.00	206.50	144.11	75.49
Equity Payment					830.37	830.37	830.37
End balance		500.00	1,150.00	2,065.00	1,441.13	754.88	0.00

The chief merit of the TIP schedule lies in the fact that at year  $t$ , the annual present values of the future cash flows from  $t+1$  until the end of the project are explicitly calculated. These values in this table will facilitate the estimation of the annual values of equity and the annual debt-equity ratios.

The balance at the end of year five is zero because we had assumed that the NPV of the project was zero. In the Appendix, we will discuss the general case. As mentioned previously, we had assumed that the correct opportunity cost of capital for an unlevered cash flow is 10%. One of the advantages of the TIP schedule is that we can obtain directly the annual values of the unlevered cash flow.

Next, I will introduce the financing arrangements. I will assume that seventy percent of the annual investments in year zero through year two will be financed by debt.

Note that this financing arrangement will result in changing annual debt-equity ratios and is different from the typical analysis where it is assumed that the debt-equity ratio is constant. Based on this financing arrangement, the loan schedule can be constructed. The cost of the debt is assumed to be 6%, and the full loan will be repaid in three equal installments from year 3 to year 5.

The complete loan schedule is shown below.

**Table 4: Loan Schedule**

	Year>	0	1	2	3	4	5
Annual Borrowings		350.00	420.00	560.00			
Beg Balance			350.00	791.00	1,398.46	959.19	493.56
Interest Accrued			21.00	47.46	83.91	57.55	29.61
Actual Debt Payment					523.18	523.18	523.18
End balance		350.00	791.00	1,398.46	959.19	493.56	0.00

In year five, the ending balance is zero because we are assuming the loan will be fully repaid.

The cash flow for the financier is shown below, and the annual present values are also calculated.

**Table 5: Annual present values in year t of the future financing cash flows from year t+1 until the end of the project**

	Year>	0	1	2	3	4	5
Financing cash flow		-350.00	-420.00	-560.00	523.18	523.18	523.18
PV of Loan, in year t		350.00	791.00	1,398.46	959.19	493.56	

Note, as expected, the annual ending balances in the loan schedule in Table 4 are equal to the corresponding annual present values in Table 5.

Now I will invoke the assumption that M & M's proposition holds in each period. That is, in each period, the value of the unlevered cash flows is equal to the levered cash flows, which consists of the cash flows to the financier and the equity holder. Thus, the

annual values of the equity are equal to the differences between the annual present value of the unlevered cash flows and the annual present value of the financing cash flow. Simply put, M & M's proposition asserts that financing does not affect the valuation. To confirm the results, see the note in Appendix B.

**Table 6: Annual present values in year t of the future equity cash flows from year t+1 until the end of the project**

	Year>	0	1	2	3	4	5
Debt			350.00	791.00	1,398.46	959.19	493.56
Total Value (TIP)			500.00	1,150.00	2,065.00	1,441.13	754.88
Equity			150.00	359.00	666.54	481.94	261.32

Based on the values in Table 6, we can calculate the annual debt-equity ratios and the annual returns to equity.

$$e_t = \rho + (\rho - d) * \frac{V_{t-1}^{Debt}}{V_{t-1}^{Equity}} \tag{5}$$

where  $e_t$  is the return to equity in year t,  $V_{t-1}^{Debt}$  is the value of the debt at the end of year t-1 (or the beginning of year t), and  $V_{t-1}^{Equity}$  is the value of the equity at the end of year t-1 (or the beginning of year t). The annual returns to equity are a function of the annual debt-equity ratios. For a detailed exposition on calculating the returns to equity, see Tham (1999).

The annual debt-equity ratios and the annual returns to equity are shown in the table below.

**Table 7: Annual debt-equity ratios and annual returns to equity.**

	Year>	0	1	2	3	4	5
Debt, Percent			70.0%	68.8%	67.7%	66.6%	65.4%
Equity, Percent			30.0%	31.2%	32.3%	33.4%	34.6%
Debt-equity ratio			2.333	2.203	2.098	1.990	1.889
Return to equity, e			19.33%	18.81%	18.39%	17.96%	17.56%

The debt-equity ratio (at the beginning of the year) varies from 2.33 in year one to 1.89 in year five, and consequently, the required annual return to equity also varies from 19.33% in year one to 17.56% in year five.

In order to verify our calculations, we can construct both the equity cash flow statement and the Equity Schedule. Using the annual returns to equity, we would expect the NPV of the cash flow statement to be zero and the ending balance in the last year of the project in the Equity Schedule to be zero.

**Table 8: Equity Cash Flow Statement**

	Year>	0	1	2	3	4	5
NCF, TIP		-500.00	-600.00	-800.00	830.37	830.37	830.37
Financing		350.00	420.00	560.00	-523.18	-523.18	-523.18
NCF, Equity		-150.00	-180.00	-240.00	307.19	307.19	307.19
Discounted		-150.00	-150.84	-169.27	183.00	155.14	131.97
<b>NPV&gt;&gt;</b>		0.00					

**Table 9: Equity Schedule**

	Year>	0	1	2	3	4	5
Annual Equity		150.00	180.00	240.00			
Beg Balance			150.00	359.00	666.54	481.94	261.32
Dividend Accrued			29.00	67.54	122.59	86.56	45.87
Equity Payment					307.19	307.19	307.19
End balance		150.00	359.00	666.54	481.94	261.32	0.00

## SECTION TWO

Based on M & M's proposition, we assume that the financing profile does not affect value. Thus, we simply use the cash flow from the Total Investment Point of View to determine the correct level of the annual tariffs and we do not need to calculate or specify the annual returns to equity.

Often, the equity holders may propose that the **initial** debt-equity ratio is the correct value to use in the calculation of the return to equity. In this case, the initial debt-equity ratio is 2.33. Based on this value, the return to equity would be

$$e = \rho + (\rho - d) \frac{V_{t-1}^{\text{Debt}}}{V_{t-1}^{\text{Equity}}}$$

$$e = 10\% + (10\% - 6\%) * 2.33 = \mathbf{19.33\%} \quad (6)$$

If the tariff for the BOT project are designed to provide an annual return of 19.33%, based on the initial debt-equity ratio, the tariff payments will be excessive. The use of the initial debt-equity ratio overstates the debt-equity ratio over the life of the project because the value of the debt is reduced over the life of the project.

## CONCLUSIONS

By relaxing many of the typical assumptions, this approach increases the flexibility in the analysis of projects that are financed on a project basis. In particular, with the construction of the cash flow from the equity point of view, the risk profile of the project for the equity holder can be explicitly assessed.

## APPENDIX A

In the previous analysis, it was assumed that the NPV of the cash flow statement from the Total Investment Point of View was zero. And the annual present values were calculated in two ways: first, using the cash flow of the TIP, and second with the TIP schedule. In practice, there is no reason for the NPV to be exactly zero. In this Appendix, I will briefly outline the general procedure for estimating the annual values of the unlevered cash flows. If the NPV of the unlevered cash flow is not zero, it means that the ending balance of the TIP schedule will not be zero. When the NPV of the unlevered cash flow is not zero, the annual present values of the TIP cash flows will not match the annual ending balances in the TIP schedule. In order to ensure that both of the methods give the same answers, a small adjustment is needed in the calculation of the annual ending balances in the TIP schedule.

If the NPV is positive, the ending balance of the TIP schedule in the last year of the project will be negative. This means that the payment to the equity holder is in excess. And similarly, If the NPV is negative, the ending balance of the TIP schedule in the last year of the project will be positive. This means that the payment to the equity holder is insufficient.

Consider the first case where the NPV of the unlevered cash flows is positive. The annual present values of the ending balance in the last year (in absolute value terms) must be calculated. If the NPV of the unlevered cash flows is positive, then the annual present values of the ending balance in the last year must be **added** to the annual ending balances in the TIP schedule. And it will be easy to verify that this adjustment will provide estimates of the annual present values of the unlevered cash flows that exactly match the values obtained by directly using the unlevered cash flows rather than the TIP schedule.

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