Was it Discrimination or the Market?  
Female Employment and the Wage Gap

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ABSTRACT

This paper quantitatively tests how much of the post-WWII evolution in employment and average wages by gender can be explained by a model where changing labor demand requirements are the driving forces. I argue that a big fraction of the original female employment and wage gaps in mid-century, and the subsequent shrinking of both gaps, are explained by labor reallocation from brawn-intensive to brain-intensive jobs favoring women’s comparative advantages in brain over brawn. I analyze the effects of an exogenous “brain-biased” technical change, increasing the relative productivity of brain-intensive to brawn-intensive production processes, on aggregate employment and wage gap trends. Initial results suggest the mechanism to be able to explain 37 to 67 percent of the rise in married female labor force participation, about 89 percent of the rise in single female labor force participation and about 53 to 62 percent of the closing wage gap, with an initially slower growth rate in average female to male wages due to selection bias. Moreover, the model, similar to the data, generates fairly steady married and single men’s labor force participation over time.

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1 Theories and Facts on Women’s Employment

This study provides evidence from United States data and develops a general equilibrium model based on the following three facts of labor demand, labor supply and wages since World War II:

1. Women’s labor force participation, aged 25 to 64, rose from only 32 percent in the 1950s to 71 percent in 2005. More specifically, married women increased their labor force participation by 46 percentage points and single women by 15 percentage points (see figure 1), while men’s labor force participation stayed fairly steady.

2. The gender wage gap, defined as average female to average male wages, changed quickly during the same period. Initially falling from close to 65 percent to a low of 57 percent in the mid 1970s, before starting to close again reaching around 77 percent by 2005 (see figure 1). While some of the initial fall in the wage gap was likely due to World War II, a stagnation is undisputable.

3. A selection bias of women into occupations with initially lower wages, and a subsequent rise of the relative returns to these occupations, coupled with a rise in women’s relative labor supply to these occupations can be clearly seen in the data (the classification of these occupations will be discussed in detail in section 2 and 5). That is, the wage gap closed for two reasons, (1) a rise in the returns to “female-friendly” occupations, and (2) a faster rise in the female to male efficiency unit labor supply to these occupations (see figure 2).

While it is a popular perception that anti-discrimination laws focused on gender equality were the main reasons behind women’s changing labor market participation and earnings, economic studies have found various other reasons played an important role in shaping women’s labor market experience, such as changes in women’s work experience, education, and occupational changes (see for example Black and Juhn, 2000; Blau, 1998; Mulligan and Rubinstein, 2005, and references therein). The main driving force in changing female employment and wages should be of particular interest to policy makers. For example, in order for the European Union to reach its 2010 Lisbon target of raising the proportion of women in the labor force from 51 to 60 percent, European Union policy makers can learn from what has driven the United States or other countries’ female
labor force convergence. Will labor protection laws need to change? Has discrimination played a mayor role? Should subsidized childcare be provided? The European Union has pledged to provide childcare for 90 percent of children aged three to the mandatory school age, and for 30 percent of children under the age of three by 2010 (Broughton, 2002). How effective will this be? What about the changing labor demand structure in countries that transition from an industrial to a service economy who saw a big rise in female employment and average wages?

This paper presents evidence from the United States and develops a general equilibrium model where women’s improved labor market experience is driven by labor demand changes. More specifically, I argue that the main factor in improving women’s labor market opportunities, and their potential wages, is the shift in labor shares away from the brawn-intensive sector. Therefore, the goal of this paper is to test the quantitative importance of labor demand changes in explaining the shrinking wage gap and the rise in female labor force participation. Note that while the study focuses on evidence from United States data, the model developed should also be able to explain cross-country differences in women’s labor market participation.

The rise of female labor force participation has been the focus of many recent macroeconomic papers. Some of these studies argue that improvements in home technology, such as the invention and marketization of household appliances or the improvements in baby formulas, enabled women to enter the labor market. For example, Greenwood et al. (2002, and references therein) develop a model where the price of durable goods, which are labor-saving in household production, falls over time. The authors find that their model is able to explain more than half the rise in female labor force participation, while an exogenously closing wage gap can only explain a small portion of the closing employment gap. Similarly, Albanesi and Olivetti (2006) develop a model where women’s comparative advantage of nurturing children and the invention of the baby formula freed women’s time to enter the labor market. They find a strong complementary in the adoption of new infant feeding practices and labor force participation of married women. While, improvements in home technology imply that women have more options in maximizing the value of their time and, therefore, can move into the labor market, theories only focused on home technological improvements do not and cannot effectively address the evolution of the wage gap over time.
A second set of studies argues that certain observed labor market changes, such as the closing wage gap or the increased returns to experience for women, are largely responsible for the rise in female employment. For example, Jones et al. (2003) argue that the shrinking wage gap, closing exogenously over time, is possibly due to falling discrimination, which in turn is the main factor in attracting women into the labor market. Olivetti (2006) develops a model where increasing returns to experience attract women into the labor market. She finds that the increased returns to experience can explain a large portion of the closing employment gap, while in her model an exogenously closing wage gap cannot. However, neither of these two studies explain why women suddenly earned higher wages or had higher returns to experience, leaving this mechanism behind the closing wage gap unexplained.

To summarize, while previous studies have been successful in explaining part of Fact 1, the rise of the female labor force, they say nothing about the closing gender wage gap beyond taking Fact 2 as given. That is they only address one aspect of the events shaping women’s labor market experience.

Recent focus has been on the effects of cultural/social or intergenerational learning on labor demand. Fernández (2007), and Fogli and Veldkamp (2007) both develop models where new generations learn from women’s past labor market experiences. As before, these models are successful in explaining parts of the rise in female labor force participation. In addition, Fogli and Veldkamp (2007) extend their theory to explain the evolution of wages, where average female wages change because of selection, meaning that the type of women that work changed in the 20th century. However, the model is unable to match the complete wage evolution, either only matching the initial stagnation or the later rise.

Note that all previously mentioned studies focus on labor supply side changes while keeping the labor demand constant. Naturally, this leaves one big unexplored fact, the changing labor demand. Two recent econometric studies analyze the effects of labor inputs in production on the gender wage gap. Wong (2006) develops a model where machine innovation of intermediate producers is channeled toward the growing sector. However, the main focus is on the effects of the
introduction of the birth control pill and the change in perceived intellectual level by employers, boosting women’s relative wages, since agents choose whether to supply intellectual abilities to a “high-tech” or physical strength to a “low-tech” sector. Through an econometric model, the importance of birth control on the rise of female wages during the last three decades is estimated. She concludes that the pill had little impact in raising women’s wages, and, in addition, skill-biased technical change had a similar impact on men’s and women’s wages and therefore, cannot explain the closing wage gap. This study differs in the determination of intellectual versus physical strength requirements, using a finer decomposition, and modeling more complex interaction between brain and brawn, which can partially explain the divergence in results. Addressing the topic from a different angle, Black and Spitz-Oener (2007) quantify the contribution of changes in specific job tasks on the closing wage gap from 1979 to 1999 for West Germany. The authors find that women have experienced an increase in non-routine analytic tasks (researching, planning, etc.) and non-routine interactive tasks (negotiating, teaching, etc.), and, in contrast to men, a marked decreases in routine cognitive (calculating, bookkeeping, etc.) and routine manual (operating and equipping machines) benefiting women’s average wages.\(^1\) Overall findings suggest that these relative changes, in quantity as well as changes in specific task prices, can explain about 41 percent of the closing wage gap for the 20 year time span in West Germany (see Table 4 in Black and Spitz-Oener, 2007, for a break down of the contribution.), which according to the authors estimates are likely driven by technological change since within occupation task changes and task changes within occupations utilizing computers were mainly driving the changes in task supply.

While these two studies estimate the effects of relative labor demand changes on the wage gap both assume an inelastic labor supply, ignoring the changing employment gap, and, as a consequence, the implication that different types of women self-select into the labor market and, therefore, shape the non-linear path of average female wages.

Undoubtedly trends in demand changes are missing from macroeconomic theory focusing on the rise of the female labor force and the shrinking wage gap. I argue that the trends in female employment and gender wage gap stem from one underlying economic process, technical change

\(^1\)For a detailed description of tasks Black and Spitz-Oener (see Table 1 in 2007).
leading to labor reallocation from a brawn-intensive to a brain-intensive sector. Quantitative results from a theoretical framework with a “brain-biased” technical change or a Hicks-neutral faster TFP growth in the brawn-intensive sector are provided in this paper. Moreover, quantitative results from a simple extension allowing for educational investment are also provided.

As changes in labor demand are the key motivation for this study, section 2 provides further evidence for the changing labor market, focusing on (1) the estimation and evolution of physical and intellectual requirements in the United States over time, (2) women’s self-selection into low-strength jobs due to physical hurdles, and (3) the effects of the changing demand for physical and intellectual attributes on female and male wage differentials. Section 3 outlines the general equilibrium model to match the labor market trends from section 2. The economy consists of two intermediate goods production, a brain-intensive and a brawn-intensive production process. These intermediate goods are aggregated by a CES production function to produce a market goods. Heterogeneous agents differ in their innate intellectual and physical abilities and therefore in their willingness to work with either technology or in the labor market at all. Households maximize consumption over market produced goods and home produced goods by allocating time between the labor market and home. Section 4 discusses the effects on labor demand, supply and wages resulting from brain-biased and Hicks-neutral technical progress. In section 5, these effects are quantified for the United States by simulating the model to match United States data targets. Initial results on married households shows that brain-biased technical change is able to replicate about 53 percent in the rise in married female labor force participation and about 40 percent in the closing wage gap, with an initially slower growth rate in average female to male wages due to selection bias. Section 6 discusses a future extension of the model - endogenizing brain-biased technical change and section 7 concludes.

To summarize, this study’s contributions are focused on providing an endogenous theory for the rise of female labor employment and the shrinking wage gap simultaneously. It provides a complementary explanation to home technological improvements, by analyzing technological changes in the market favoring brain over brawn. While improvements in home technology push women into the labor market by freeing their time, labor reallocation and brain-biased technical change pull
women into the labor market by raising the opportunity cost of staying at home - making these
two hypothesis complementary. However, in contrast to home technology improvements, techno-
logical improvement in the marketplace also provides an explanation for the shrinking wage gap.
In addition, while the empirical results are specific to the United States, the theoretical framework
suggests that the model should be able to explain cross-country differences as suggested by Roger-
son (2005). He notes that the change in relative employment of women and the aggregate service
share (I argue this to be a brain-intensive sector in section 6) between 1985 and 2000 are highly
correlated at 0.82, concluding that countries which added the most jobs to the service sector also
closed the employment gap the most.

2 The Expanding Female Friendly Labor Sector

To explore the correlation between the rise in female labor force participation and changes in
labor demand I focus on the relative demand and supply of two types of labor inputs, intellect and
physical strength (henceforth brain and brawn, respectively). This study starts from the premise
that women have on average less brawn then men. One well document area where women are barred
from certain occupations because of physical strength requirements is the military. For example, an
article in “BBC News Online” (BBC News Online, 2002) notes that starting in 2002 women were
barred from front-line combat since they failed to pass the required physical test, where “Soldiers
under 30 had to carry 20kg of equipment and their rifle while running a mile and a half in 15
minutes, as well as carrying a colleague for 50 yards.” Similarly, a study commissioned by the
Israeli military (CNN.com, 2003) found that “that most women are not able to lift the minimum
amount required of combat soldiers, 110 pounds (49 1/2 kilograms). It also said most women could
not complete military treks, which typically involve carrying heavy gear, of more than 12 miles (19
kilometers). Male soldiers can be required to march more than twice that distance.” Accepting that
women and men have similar levels of brain, men have a comparative advantage in brawn-intensive
occupations. However, changes in technology shifting labor demand toward low-brawn occupations
diminishes men’s comparative advantage in the labor market.
2.1 Estimating Brain and Brawn

In order to obtain predictions on the evolution of brain and brawn requirements, I obtain brain and brawn estimates by U.S. census occupation and industry classifications from the Dictionary of Occupational Title (DOT) 1977. This DOT survey set is particularly useful due to three facts: (1) it is readily available in computer format, (2) it has been merged with the 1971 CPS allowing for civilian employment population weighted results, and (3) it lies mid-way through the sample (the 1970s). To estimate brain and brawn levels over time across industries and gender I use factor analysis as Ingram and Neumann (2006). Factor analysis is a technique to reduce a large number of variables, called characteristics, within a dataset to a few unobserved random variables, called factors.

The DOT reports 38 job characteristics for over 12,000 occupations, documenting (1) general educational development, (2) specific vocational training, (3) aptitudes required of a worker, (4) temperaments or adaptability requirements, (5) physical strength requirements, and (6) environmental conditions. Characteristics measure certain skills required of a worker in a given occupation. For example, general educational development measures a worker’s formal and informal educational attainment to preform his/her job effectively by rating reasoning, language, and mathematical developments. Where each levels is primarily based on curricula taught in the United States, such that the highest mathematical level requires advanced calculus, while the lowest level only requires basic operations, such as adding and subtracting two-digit numbers. Specific vocational preparation is measured in the number of years a typical employee requires to learn the job tasks required to perform at an average level on the job. Eleven aptitudes required of a worker (e.g., general intelligence, motor coordination, numerical ability) are rated on a five point scale, with the first level being the top 10 percent of the population and the fifth level compromising the bottom 10 percent of the population. Ten temperaments required of a worker are reported in the 1977 DOT, rather than rating the level only the temperament type is reported. An example of a temperament is the adaptability to influence people in their opinions or judgments. Physical requirements include a measure of strength required on the job, rated on a five point scale from sedentary to very heavy.

\textsuperscript{2}Data is available, including documentation, from the Inter-university Consortium for Political and Social Research (ICPSR).
and the presence or absence of tasks such as climbing, reaching, or kneeling. Lastly, environmental conditions measure a workers exposure (presence or absence) to environmental conditions, such as extreme heat, cold, and noise (see Vijverberg and Hartog, 2005, for method on converting DOT scales to scales for use in estimation procedures).

These 38 characteristics capture the heterogeneity across jobs and workers. While these characteristics measure different specific requirements they can be grouped into broader categories of skills in terms of their common underlying dimensions, thereby reducing the dimensionality of heterogeneity. To determine whether such characteristics as language, mathematical and reasoning development, numerical, verbal aptitude, etc. can all be explained by some functional form of one factor - intellectual ability - I employ factor analysis. Factor analysis can be used to study the relationship of a set of dependent variables to uncover the independent variables and the functional form of these independent variables that determines the dependent variables observed in the data. In the general specification the characteristics, $C_i$, are modeled as linear combinations of the independent variables or factors, $F_i$ plus an error term $\epsilon_i$,

$$C_i = \mu + \Lambda F_i + \epsilon_i \quad \text{for } i=1,\ldots, N,$$

where $N$ equals the number of occupations, $C_i$ is the vector of characteristics ($38 \times 1$), $\mu$ is the vector of characteristic means, $\Lambda$ is a vector of coefficients to be estimated called factor loadings, $F_i$ is a vector of the factors (here $3 \times 1$), and $\epsilon_i \sim N(0, \Sigma)$ is the uncorrelated error vector, with $\Sigma$ being the diagonal variance covariance matrix. To obtain population representative estimates the occupations in the DOT need to be weighted. As the DOT itself does not record the number of each worker in the economy for a given job, in order to weight DOT jobs, the 1971 CPS merge is used. In this particular dataset the *Committee on Occupational Classification and Analysis of the National Academy of Sciences funded by the Department of Labor and the Equal Employment Opportunity Commission* merged the 12,431 1977 DOT jobs to 7,289 unique occupation-industry pairs from the 1970 U.S. Census, providing 1971 CPS weights of the civilian labor force for all jobs. The reduction from 12,431 to 7,289 jobs occurs since the DOT provides more detailed occupation listing, for example while there is only one “waiter/waitress” category in the census classification the
DOT has multiple, such as “waiter/waitress formal”, “waiter/waitress, Head”, “waiter/waitress, take out.”

Since only information on the characteristics is available, we use this information to estimate both, Λ and \( F_i \) from

\[
E(C - \mu)(C - \mu)' = \Lambda E(FF') \Lambda' + \Sigma,
\]

that is the covariance in the 38 characteristics can be explained by a reduced number of factors \( F_i \), where \( C = [C_1C_2...C_N] \) and \( F = [F_1F_2...F_N] \). However, it is clear that \( \Lambda \), \( E(FF') \), and \( \Sigma \) are not separately identifiable from this expression. Therefore, factor analysis generally assumes factors to follow a standardized normal distribution, which allows for the identification of \( \Sigma \). To separately identify \( \Lambda \) and \( E(FF') \) additional restrictions need to be placed. Commonly the covariance between factors is set to zero, that is,

\[
E(FF') = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & & \ddots & \\ 0 & \cdots & 1 \end{bmatrix}
\]

allowing us now to identify both \( \Lambda \) and \( \Sigma \), which is diagonal by assumption, separately. That is in this specification characteristics are a function of all factors. In practice, the first factor estimate will explain the maximum possible covariance between the characteristics. The second factor is estimated to explain the maximum covariance remaining, and so on. In practice we could estimate 38 factors, in which case we would need 38 factors to explain the covariance between all the dependent variables, the characteristics. However, in this case three factors explain most of the characteristics covariance structure. Ingram and Neumann use the DOT 1991 with over 53 characteristics, primarily expanded by detailing physical and environmental characteristics (using virtually all other characteristics) to estimate a total of four factors: (1) intelligence, (2) clerical skill, (3) gross motor skill, and (4) ability to deal with physically and hazardous work. However, as in Ingram and Neumann (2006) the first factor is positively related to intellectual characteristics and negatively to coordination and physical characteristics reported in the DOT, which makes it difficult to interpret the factors consistently. Therefore, I reestimate the factors by assuming,
similar to Ingram and Neumann (2006), factors to be correlated except, for identification purposes, job characteristics that explain one factor are restricted to not explain another factor. That is, mathematical development only explains a job’s intellectual requirements directly, while it is only informative on the job’s physical requirements through the correlation of the aggregate intellectual and physical factor. Table A.1 in Appendix A provides the classification of characteristics across factors as well as the factor loading coefficients, which are used in determining factor values for each occupation-industry combination present in the 1971 CPS. Given the grouping of characteristics and the estimates of factor loadings I call the three factors brain, coordination/motor and brawn. The scatter plot in figure 3 illustrates the over 12,000 combinations of brain and brawn factors by occupations in the United States from the 1977 DOT. Moreover, 1971 CPS labor shares by subgroups of brain and brawn combinations are provided. In 1971, 50.08 percent of the population worked in occupations requiring more brain than brawn, with 42.20 percent working in occupations requiring above average brain and below average brawn, while 33.86 percent of the population worked in an occupation requiring above average brawn and below average brain.

2.2 The Evolution of Brain and Brawn in the United States

The above estimation results in factor requirements by occupation-industry pairs for 1971. To compute aggregate factor demand changes in the United States over time, occupation-industry pairs are aggregated using weights from the civilian labor force of the United States Census survey prior to 1968 and of the CPS thereafter (consistent data are obtained from the IPUMS-USA (Ruggles et al., 2004) and the IPUMS-CPS project (King et al., 2004)).

Note, that using a single DOT survey to determine job requirements, implies that the specific job factor requirements did not change over the last 5 decades, that is a craftsman utilized the same brawn level in 1950 as in 2005. Ergo, all trends pictured are due to changes in the composition

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3 Motor skills do not change much over time (see figure 4) and do not differ much between gender and are, therefore, omitted for the remained of this study.

4 The IPUMS projects provides a consistent 1950 Census classification of occupations and industries over the years, which is used in merging 1977 DOT brain and brawn factors by 3-digit census occupation and 3-digit industries.

5 Since factors could only be estimated for occupation-industry pairs represented in the 1971 CPS, to achieve the greatest number of possible matches between factors and Census/CPS dataset over the years factors are first matched by three-digit occupation-industry pairs, than by three-digit occupation/one-digit industry pairs, than by one-digit occupation-industry pairs, and lastly by one-digit occupations.
(mix) of occupations within the economy, and the rise in brain and fall in brawn requirements might possibly be greater than shown. Figure 4 provides the aggregate factor deviations from their means which are normalized to zero in 1950. While the “third” factor supply, coordination, remains fairly steady over time, the brain supply steadily increases with the brawn supply mirroring the opposite trend (steadily decreasing). This trend in the rise in brain supply and the fall in brawn supply is what I term brain-biased technical change (see section 1).

Lastly, figure 5 shows brain and brawn deviations by gender over time, the selection of women into occupations with low brawn requirements is apparent. Note that this bias toward low brawn occupations can be either due to self selection through ability or due to employer “discrimination” given women’s lower innate brawn levels. Also note the rise in brain supply, with women surpassing men by the end of the century and the fall in men’s brawn supply.

2.3 Wages as a Function of Brain and Brawn

The evidence so far underlines the strong correlation mentioned previously between the rise of female employment and the rise of brain-intensive occupations. While, the data points to a clear selection of women into low-brawn jobs, the effects of this labor reallocation on the wage gap still needs to be determined.

As mentioned previously Black and Spitz-Oener (2007) quantify the contribution of changes in specific job tasks, due to technological change, on the closing wage gap from 1979 to 1999 for West Germany. The authors suggest that as much as 41 percent of the closing wage gap can be explained by task supply and price changes. To determine the effect of brain and brawn changes on the wage gap I repeat this exercise for the United States given my estimation of brain and brawn. Note, however, that contrary to Black and Spitz-Oener (2007) brain and brawn requirements are only changing in the aggregate economy due to changes in occupation and industry mix rather than due to changes within a given occupation.

The contribution of changes in relative factor supplies by gender and relative factor prices in
the economy on the wage gap are computed from the following identity,

\[
(w_{m,T} - w_{f,T}) - (w_{m,0} - w_{f,0}) = \sum_j r_j (F_{j,m,T} - F_{j,f,T}) + \sum_j (F_{j,m} - F_{j,f}) (r_{j,T} - r_{j,0}),
\]

where subscript 0 denotes the base year, \(w_{g,T}\) is the average log wage of gender \(g\) at time \(T\), \(r_j\) is the return of factor \(j\), \(F_{j,g}\) is the average supply of factor \(j\) by gender \(g\). Averages without time subscripts are averages between the two years, base 0 and ending \(T\). Unlike Black and Spitz-Oener (2007), factor returns are not allowed to vary across gender, since I argue women’s and men’s wages only differ because of their relative brain and brawn supplies (estimates by allowing factor returns to differ across gender are comparable). Factor demands by gender can be computed from the brain and brawn estimates using U.S. Census and CPS weights over time. To compute factor returns a log-linear wage regression on standard explanatory variables, such as age, education, plus an individual’s brain, brawn and coordination factor supplies, is estimated. The resulting coefficients on brain and brawn are taken as a proxy of factor returns (see Appendix A Table A.2 for coefficient estimates). Table 2 provides a breakdown of the contributions on the closing wage gap of relative quantity and price changes by factor between a select number of time periods,

<table>
<thead>
<tr>
<th>Percent Contribution</th>
<th>1950-1980(^a)</th>
<th>1980-2005</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brain Supply</td>
<td>-0.91</td>
<td>13.58</td>
</tr>
<tr>
<td>Brain Prices</td>
<td>1.05</td>
<td>2.39</td>
</tr>
<tr>
<td>Brawn Supply</td>
<td>11.07</td>
<td>-0.19</td>
</tr>
<tr>
<td>Brawn Prices</td>
<td>-47.99</td>
<td>13.42</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>-36.79</strong></td>
<td><strong>29.20</strong></td>
</tr>
</tbody>
</table>


\(^a\)Wage gap widened during this period

Changes in brain and brawn over time can explain about one-third of the changes in female
to male average wages. As the gender gap widened from 1950 to 1980 the total contribution was negative, that is 36.79 percent of the widening wage gap can be explained by changes in factor returns and supply where this is mainly due to rising returns to brawn during this period. Note that during this time period changes in brawn supply across gender actually prevented the gap from widening even further. During the second period, 1980 to 2005 the gap closed considerably, with changes in relative brain supply, a faster rise of women’s to men’s brain supply, changes in brawn prices, and a fall in the returns to brawn explaining most of this trend.

Therefore, given all the above facts, I argue that beginning in the 1950s, due to the rise of the brain-intensive occupations, women entered the labor market and their average wages improved since the emphasis in the production process shifted to brain inputs playing to their comparative advantage. A model consistent with these facts: (1) the rise of a brain-intensive sector, (2) the rise in women’s participation, and (3) a rise in average female wages mainly driven by the brain supply and brawn price changes, is developed in the remainder of this paper.

3 Brain and Brawn: a General Equilibrium Model

Time is discrete $t = 1, 2, 3, \ldots$, and the economy consists of a unit measure of married households.$^6$ The economy consists of two intermediate production sectors, one termed the “new” and the other the “old” sector, both of which use brain and brawn inputs in the production processes. The two sectors produce intermediate goods that are aggregated to a final market good, which is consumed by households. Households can choose to work in the labor market or the home and substitute consumption between market and home produced goods.

3.1 Household Maximization

Households solve a static nested CES utility function with goods produced in the marketplace $c$ and at home, $c_h$. Omitting a labor-leisure choice, agents can divide their time between mar-

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$^6$Single households are omitted here, since they add no additional dynamics to the analytical model. However, the model simulation will incorporate both single and married households, showing the ability of the model to explain both single and married women’s labor market behavior.
ket and home production. For simplicity all households maximize a common utility function.\(^7\) Consequently, the household utility function, omitting time subscripts, is,

\[
U(c, c_h) = \ln\left((c^\nu + c_h^\nu)^{1/\nu}\right)
\]

where the substitution between market and home goods equals \(\epsilon_\nu = \frac{1}{1-\nu}\). In a static environment a married household \(k\) maximizes this utility function subject to the household production function, time and budget constraints,

\[
\begin{align*}
\max_{\{c_k, c_h, k, \ell_f, k, \ell_m, k\}} & \quad U(c_k, c_h, k) \\
\text{s.t.} & \quad c_k \leq \ell_f, k \omega_f, k + \ell_m, k \omega_m, k, \\
& \quad c_{h, k} = A_h (1 - \ell_f, k + 1 - \ell_m, k) \\
& \quad \ell_f, k = \{0, 1\} \text{ and } \ell_m, k = \{0, 1\}.
\end{align*}
\]

Household production is a linear function of time where the husband’s and wife’s labor time are perfect substitutes. Also note, that given evidence on the intensive and extensive margin of labor supply (see appendix figure B.1), it is assumed that agents can only work either full-time or not at all in the labor market, \(\ell_{g, k} = \{0, 1\}\) for gender \(g\) of household \(k\). Lastly, agents can earn the wage \(\omega_{g, k} = \psi(b_{g, k}, r_{g, k})\), a function of an agent’s innate brain and brawn abilities, in the labor market. To determine this functional form it is necessary to first describe the firm’s problem. Note that agents are uniquely identified by the gender-household \(g, k\) subscript. Given perfect substitution in home production households specialize with the higher wage earner entering the labor market first.

The primary wage earner of household \(k\) works in the market if

\[
\omega_{1, k} > \frac{2A_h}{T_{\ell_{1, k}}} \tag{5}\]

\(^7\)Alternatively, households could maximize a weighted utility function. However, this only complicates the model without adding further insight given that this study abstracts from marriage and divorce.

\(^8\)Single agents have a similar labor supply threshold, which is \(\omega_{g, k} > A_h\).
The secondary wage earner enters the market if and only if the above conditions are satisfied in addition to

\[
\omega_{2,k} > \left( \frac{\omega_{1,k} + A_h^{\nu}}{\nu} - \omega_{1,k} \right) \frac{1}{\nu}
\]

(6)

where \( T_{\ell_{g,k}} \) are the thresholds determining the primary and secondary workers’ labor supplies.

### 3.2 Production Process

There are two intermediate goods producers, one termed the “new”, \( n \), the other the “old”, \( o \), sector. Both sectors use brain and brawn in a linear production function, that is

\[
Y_{j,t} = \alpha_j B_{j,t} + (1 - \alpha_j) R_{j,t}, \quad \text{for } j = n, o.
\]

\( B_{j,t} \) and \( R_{j,t} \) are the aggregate quantities of brain and brawn sector \( j \) hires at time \( t \), while \( \alpha_j \) is the productivity of brain inputs in production and \((1 - \alpha_j)\) the productivity of brawn inputs, assumed fixed over time. I assume that the new sector can produce more output with brain than brawn compared to the old sector, that is \( \alpha_n > \alpha_o \). The two intermediate goods are than aggregated through a CES aggregate production function to produce the final market good,

\[
Y_t = \left( \lambda_n (A_{n,t} Y_{n,t})^\phi + \lambda_o (A_{o,t} Y_{o,t})^\phi \right)^{1/\phi},
\]

where \( A_{j,t} \) is sector \( j \)’s total factor productivity and \( \epsilon_\phi = \frac{1}{1-\phi} \) is the elasticity of substitution between new and old sector goods, and \( \lambda_n + \lambda_o = 1 \) are production shares. All markets are competitive, the number of firms in each sector is normalized to one, intermediate goods are priced at \( p_{j,t} \) and the final good price is normalized to one.

Letting \( w_{b_{j,t}} \) and \( w_{r_{j,t}} \) be the factor prices of brain and brawn in each sector \( j \), the first order condition from the intermediate goods producers are,

\[
w_{b_{j,t}} = p_{j,t} \alpha_j, \quad (7)
\]
and

\[ wr_{j,t} = p_{j,t}(1 - \alpha_j). \]  

(8)

Since any agent, when choosing to supply his/her time to the labor market, supplies both brain and brawn, firms will always use both inputs, rather than hiring only the cheaper/more productive input. That is firms hire an agent’s efficiency units of labor \( l_{g,k} = \alpha_j b_{g,k} + (1 - \alpha_j)r_{g,k} \). Relative factor returns will differ in the two intermediate goods markets with

\[ w_j = \frac{w_{b,j,t}}{w_{r,j,t}} = \frac{\alpha_j}{1 - \alpha_j}. \]

The price of intermediate goods is determined by minimizing the cost of producing the final market good. Where relative price equals,

\[ p = \frac{p_{n,t}}{p_{o,t}} = \frac{\lambda_n}{\lambda_o} \left( \frac{A_{n,t}}{A_{o,t}} \right)^{\phi} \left( \frac{Y_{n,t}}{Y_{o,t}} \right)^{\phi-1}. \]  

(9)

That is relative prices are a function of total factor productivity as well as intermediate goods supplied. Note that \( \frac{Y_{n,t}}{Y_{o,t}} = \frac{L_{n,t}}{L_{o,t}} \), where \( L_{j,t} = \alpha_j B_{j,t} + (1 - \alpha_j)R_{j,t} \) are labor efficiency units in sector \( j \), obtained by aggregating individual agents’ labor efficiency unit supplies over the distributions of brain and brawn. Using this first order condition, and the aggregate production function, demand for both intermediate goods per one unit of aggregate good are,

\[ y_{j,t} = \frac{Y_{j,t}}{Y_t} = (A_{j,t})^{\phi-1} \left( \frac{\lambda_j}{\lambda_{j,t}} \right)^{\epsilon_j} \left[ \lambda_n^{\epsilon_n} \left( \frac{A_{n,t}}{p_{n,t}} \right)^{\epsilon_n^{-1}} + \lambda_o^{\epsilon_o} \left( \frac{A_{o,t}}{p_{o,t}} \right)^{\epsilon_o^{-1}} \right]^{-1/\phi}, \]  

(10)

where the last term is the unit cost of the aggregate production.

Lastly, since markets are competitive, it is assumed that agents receive all rents from production and, therefore, factor returns follow from the first order conditions of intermediate goods producers (7) and (8), note that the returns to brain and brawn differ across intermediate sectors. That is a rise in the price of an intermediate goods is captured in a one-to-one rise of that good’s factor returns.
3.3 Wages and the Distribution of Brain and Brawn

We can now explicitly state an agent’s wage, $\omega_{g,k}$, which is determined by his/her innate brain and brawn ability. From the firm’s problem it follows that $\omega_{g,k,t} = \max \{p_{n,t}(\alpha_n b_{g,k} + (1 - \alpha_n) r_{g,k}); p_{o,t}(\alpha_o b_{g,k} + (1 - \alpha_o) r_{g,k})\}$. Moreover, brain and brawn are jointly distributed $(b_k, r_k) \sim A_g(b, r)$ with differing distributions by gender. Since, the premise of this study is the lack of women’s brawn, the two gender distributions $A_m(b, r)$ and $A_f(b, r)$, only differ in their distribution of brawn $r_g \sim R_g$. Consequently, the distribution of brain, $b \sim B$, and the correlation of brain and brawn, $\rho$ are identical across gender. Additionally men and women are connected through marriage, therefore, any given household has a four-dimensional joint distribution over female brain, female brawn, male brain, and male brawn. That is there are potentially four correlation parameters to determine the joint distribution of a household. For simplicity it is assumed that men and women are matched randomly in all dimension (correlation equals zero) but brain, where men and women’s brain quantities have the correlation coefficient $\rho_b > 0$ given evidence on assortive matching in educational attainment (see Fernández et al., 2005).

4 Dynamics of TFP Changes on Labor Demand, Supply and Wages

This study argued in section 2 that labor has moved from brawn-intensive toward brain-intensive production. In this set-up, the old sector is clearly more brawn-intensive than the new sector. Any technical change, defined as a change in $A_{n,t}, A_{o,t}$, mimicking the movement from brawn-intensive to brain-intensive occupations must increase the relative demand for the new sector goods compared to the old. I analyze the changes in labor demand, supply and wages resulting from a “one time” change in relative factor productivity $A_{n,t}/A_{o,t}$. The dynamics of a steady change in relative technology parameters can simply be deduced by allowing this one time change to occur repeatedly, where $A_{j,t} = A_{j,t-1}(1 + \gamma_j)$ with the sector technology growth rates $\gamma_j$ satisfying $\gamma_n \neq \gamma_o$. 
4.1 TFP Growth on Relative Intermediate Goods and Labor Demand

The relative intermediate goods demand follows from the unit labor demands (10),

\[ \frac{Y_{n,t}}{Y_{o,t}} = \left( \frac{A_{n,t}}{A_{o,t}} \right)^{\epsilon_{\phi} - 1} \left( \frac{\lambda_{n}P_{o,t}}{\lambda_{o}P_{n,t}} \right)^{\epsilon_{\phi}} \].

Taking the derivative with respect to \( \frac{A_{n,t}}{A_{o,t}} \), ceteris paribus results in

\[ \frac{\partial Y_{n,t}}{\partial A_{n,t}} = (\epsilon_{\phi} - 1) \left( \frac{A_{n,t}}{A_{o,t}} \right)^{\epsilon_{\phi} - 2} \left( \frac{\lambda_{n}P_{o,t}}{\lambda_{o}P_{n,t}} \right)^{\epsilon_{\phi}} \],

which is positive if \( \epsilon_{\phi} > 1 \), that is the two goods are substitutes in the aggregate production process and it is negative if \( \epsilon_{\phi} < 1 \), that is the two intermediate goods are complements. An example of the first case is where a sector of brain-intensive occupations are substitutes with a sector of brawn-intensive occupations. Through brain-biased technical change, a rise in \( \frac{A_{n,t}}{A_{o,t}} \), brain-intensive occupations become more productive (this is similar to the concept of skill-biased technical change in that literature) shifting labor shares toward the brain-intensive sector. An example of the second case is where the new sector is the brain-intensive service sector (see figure 5) and the brawn-intensive industrial sector is the old sector. Through faster relative TFP growth in the industrial sector, a fall in \( \frac{A_{n,t}}{A_{o,t}} \), and complementarity in the aggregate good, the relative demand for the service sector output will rise.\(^9\) In this paper I choose to estimate the first scenario, since a purely brain-biased technical change will better match the change in brain and brawn supplied over time. However, the second scenario is of interest, since as will be explained in section 6, this classification of aggregate sectors will allow me to endogenously model brain-biased technical change in future research, providing a theory to why the production process shifted from brawn to brain.

Note that the effects of a change in relative factor productivity apply for the relative labor demand of efficiency units of brain-intensive to brawn-intensive occupations as well, since \( Y_{j,t} = L_{j,t} \). Therefore, a rise in relative TFP of the new technology sector will increase relative labor demand.

\(^9\)Factual evidence from studies on price elasticity of services (see Ngai and Pissarides, 2005, and references therein) support a complimentary between services and industrial goods.
efficiency units if $\epsilon_\phi > 1$. While, a rise in relative TFP of the old technology sector will increase relative labor demand efficiency units of the new technology sector if $\epsilon_\phi < 1$. Graphically (see figure 6), the demand for labor shifts out, the relative quantity of new to old labor efficiency units rises, and the relative prices $\frac{p_{n,t}}{p_{o,t}}$ rise as long as an outward shift in labor supply does not offset the increase in labor demand. Since, from the relative pricing equation (9), a rise in $\frac{L_{n,t}}{L_{o,t}}$ lowers relative prices since $\phi - 1 < 0$, given $\phi < 1$.

4.2 Labor Supply Decision

At the equilibrium factor wages, a change in relative TFP has no effect on $T_{t_{g,k}}$ since both thresholds (5) and (6) are independent of either TFP. Therefore, the relative labor supply does not shift and relative goods prices rise. However, a rise in relative goods prices leads to a change in factor returns, normalizing $p_{o,t} = 1$, results in a rise of factor returns in the new sector. That is, $wb_{n,t}$ and $wr_{n,t}$ rise since they are both a function of $p_{n,t}$ from (7) and (8), while $wb_{o,t}$ and $wr_{o,t}$ remain the same. This implies that the returns to brain $wb_t = wb_t(wb_{n,t}, wb_{o,t})$ rise while the returns to brawn $wr_t = wr_t(wr_{n,t}, wr_{o,t})$ fall in the aggregate economy.

A change in factor returns will change the type of person who enters the labor market. The threshold for the primary worker $T_{t_{1,k}} = 2A_b$ does not change. However, if the agent earns higher wages in the new sector his/her wage increases, that is the right hand side rises. Therefore, primary workers with relatively high brain to brawn levels, that previously stayed out of the labor market due to low wages $\omega_{1,k}$, might now enter the labor market.

An agent works in the new sector if

$$\frac{b_{g,k}}{r_{g,k}p_{o,t}(1 - \alpha_r) - p_{n,t}(1 - \alpha_b)} > 1.$$ 

The numerator is the difference in potential earnings on his/her brain ability between the new to the old sectors, and the denominator is the difference in potential earnings between his/her brawn ability in the old and new sector. If this ratio is greater than one, i.e. the additional returns to brain in the new sector are greater than the additional returns to brawn in the old sector, the agent chooses to work in the new sector. To clarify, assuming that $\alpha_b = 1$ and $\alpha_r = 0$, that is only brain
matters in the new sector and only brawn matters in the old sector, the above equation reduces to

\[ \frac{b_{g,k}}{r_{g,k} p_{o,t}(1 - \alpha_r)} p_{n,t} \alpha_b > 1, \]  

(11)

where an agent works in the new sector if his/her potential wage \( p_{n,t} \alpha_b b_{g,k} \) is greater than his/her potential wage in the old sector \( p_{o,t}(1 - \alpha_r)r_{g,k} \).

To illustrate the effects of a rise in relative prices on the secondary workers labor supply decision the following section elaborates on the dynamic effects of a simplified version making the following assumptions:

1. The new sector only utilizes brain \( \alpha_n = 1 \) and the old sector only utilizes brawn \( \alpha_n = 0 \);

2. Brain and brawn are independently uniformly distributed with \( B_g \sim [B_l, B_h] \) and \( R_g \sim [R_g, \overline{R}_g] \) for gender \( g = \{m, f\} \), where \( \overline{R}_g = \overline{R} + x_g, R_g = R + x_g \) and \( x_m > x_f \geq 0 \), the only difference between men and women is the mean brawn level;

3. Market goods and home production are perfect substitutes \( \nu = 1 \).

Note that assumption (3) implies that all agents work if \( \omega_{g,k} > A_h \), making the primary and secondary worker and the married and single household decisions identical. That is imperfect substitution between market and home produced goods, adds an interdependence of spouse’s wages to the labor supply decision, making a secondary worker of a high primary earner less likely to work. Agents work in new sector if the inequality in (11) is satisfied.

Suppressing time subscripts, the labor force participation, \( LFP_g \), of single agent of gender \( g \) is,

\[ LFP_g = \int_{\frac{\overline{R}}{\nu}}^{\overline{A}_b} \int_{\frac{b}{\nu}}^{\overline{A}_b} a_g(b, r) \, db \, dr + \int_{\frac{\overline{A}_b}{\nu}}^{\overline{A}_b} \int_{\frac{b}{\nu}}^{\overline{A}_b} a_g(b, r) \, db \, dr, \]

where \( a_g(b, r) \) is the joint probability density function. The first term are all agents that work in the new sector given home productivity and factor returns, and the second are all remaining working agents, i.e. agents that work in either sector. To not trivialize the results it is assumed
that $B < \frac{A_h}{p_n}$ and $R_g < \frac{A_h}{p_o}$, that is some agents will not work in the new sector and/or old sector. Given the special distributional assumptions labor force participation is

$$LFP_g = \frac{(B - \frac{A_h}{p_n}) (\frac{A_h}{p_o} - R_g)}{(B - B) (R - R)} + \frac{R_g - \frac{A_h}{p_o}}{R - R},$$

where $LFP_m > LFP_f$. To proof this we can substitute $R_g = \overline{R} + x$, and $R_g = \overline{R} + x$, and take the derivate with respect to $x$,

$$\frac{\partial LFP_g}{\partial x} = \frac{\frac{A_h}{p_n} - B}{B - B} \frac{1}{(R - R) (R - R)} > 0.$$

However, as the returns to brain rise, *ceteris paribus*, the employment gap will shrink, formally,

$$\frac{\partial LFP_g}{\partial p_n} = \frac{\frac{A_h}{p_n} (\frac{A_h}{p_o} - R_g)}{(R - R) (B - B)},$$

where $\frac{\partial LFP_f}{\partial p_n} > \frac{\partial LFP_m}{\partial p_n}$, since

$$\frac{\partial^2 LFP_g}{\partial p_n \partial x} = -\frac{A_h}{p_n^2} \frac{1}{(B - B) (R - R)} < 0.$$

To summarize, the increased demand for low-brawn occupations and rise of the returns to these occupations, resulting from sectoral reallocation or brain-biased technical change, lead to a shrinking of the gender employment gap, given women’s comparative advantage in brain.

### 4.3 Wage Gap Evolution

The wage gap is defined as average female to average male wages, in terms of average factor supplies to each sector,

$$\overline{w}_{f,t} = \frac{\pi_{f,t} \frac{p_{n,t}}{p_n} (\alpha_n \overline{B}_{f,n,t} + (1 - \alpha_n) \overline{R}_{f,n,t}) + (1 - \pi_{f,t}) (\alpha_o \overline{B}_{f,o,t} + (1 - \alpha_o) \overline{R}_{f,o,t})}{\pi_{f,t} \frac{p_{n,t}}{p_n} (\alpha_n \overline{B}_{f,n,t} + (1 - \alpha_n) \overline{R}_{f,n,t}) + (1 - \pi_{f,t}) (\alpha_o \overline{B}_{f,o,t} + (1 - \alpha_o) \overline{R}_{f,o,t})}$$

$$\overline{w}_{m,t} = \frac{\pi_{m,t} \frac{p_{o,t}}{p_o} (\alpha_n \overline{B}_{m,n,t} + (1 - \alpha_n) \overline{R}_{m,n,t}) + (1 - \pi_{m,t}) (\alpha_o \overline{B}_{m,o,t} + (1 - \alpha_o) \overline{R}_{m,o,t})}{\pi_{m,t} \frac{p_{o,t}}{p_o} (\alpha_n \overline{B}_{m,n,t} + (1 - \alpha_n) \overline{R}_{m,n,t}) + (1 - \pi_{m,t}) (\alpha_o \overline{B}_{m,o,t} + (1 - \alpha_o) \overline{R}_{m,o,t})},$$
where $\overline{B}_{g,j,t} = E\left( b_{g,k} \mid \frac{b_{g,k}}{r_{g,k} \rho_{n,t}(1-\alpha_r)} > 1 \land (\omega_{g,k} > \omega_{g^*,k} \lor \omega_{g,k} b_k > T_{t_2,k}) \right)$, that is the average brain level conditional on the working population of gender $g$ in sector $j$ at time $t$ (where $g^*$ denotes the spouse). Similarly, $\overline{R}_{g,j,t}$ is the average brawn conditional on the working population of gender $g$ in sector $j$ at time $t$. While $\pi_{g,t}$ is the fraction of working agents of gender $g$ working in brain-intensive occupations, the new sector.

Then, ceteris paribus, a rise in relative price results in a closing wage gap if

$$\frac{\pi_{f,t}}{1 - \pi_{f,t}} \frac{\overline{w}_{f,n,t}}{\overline{w}_{f,o,t}} > \frac{\pi_{m,t}}{1 - \pi_{m,t}} \frac{\overline{w}_{m,n,t}}{\overline{w}_{m,o,t}},$$

meaning a greater fraction of women work in brain-intensive occupations and their average wage is relatively higher in the new sector than the old sector compared with men. However, this ignores any selection bias of different types of agents into the labor market.

If lower ability agents enter the new sector with a rise in relative prices $\overline{B}_{f,n,t}$ will fall, as will $\overline{B}_{m,n,t}$, and potentially $\overline{R}_{f,n,t}$ and $\overline{R}_{m,n,t}$. I return to the simplified example from above to illustrate these effects on the average brain supplies further.

Suppressing time subscripts, the sector specific labor force participation is simply,

$$\pi_g = \int \frac{A_h}{p_o} \int \frac{A_h}{p_n} a_g(b,r) \, db \, dr + \int \frac{A_h}{p_o} \int \frac{p_n}{p_o} r_{g,k} a_g(b,r) \, db \, dr.$$

While the mean brain and brawn level of gender $g$ equals,

$$\overline{B}_g = \frac{\int \frac{A_h}{p_o} \int \frac{A_h}{p_n} b_{g,k} a_g(b,r) \, db \, dr + \int \frac{A_h}{p_o} \int \frac{p_n}{p_o} r_{g,k} b_{g,k} a_g(b,r) \, db \, dr}{\int \frac{A_h}{p_o} \int \frac{A_h}{p_n} a_g(b,r) \, db \, dr + \int \frac{A_h}{p_o} \int \frac{p_n}{p_o} r_{g,k} a_g(b,r) \, db \, dr},$$

and

$$\overline{R}_g = \frac{\int \frac{A_h}{p_o} \int \frac{p_n}{p_o} r_{g,k} a_g(b,r) \, db \, dr}{\int \frac{A_h}{p_o} \int \frac{A_h}{p_n} a_g(b,r) \, db \, dr}.$$
Using these identities, we can rewrite the gender gap to,

\[
\frac{w_{f,t}}{w_{m,t}} = \frac{p_{n,t} \tilde{B}_{f,t} + \tilde{R}_{f,t}}{p_{n,t} \tilde{B}_{m,t} + \tilde{R}_{m,t}} \frac{LFP_{m,t}}{LFP_{f,t}},
\]

(12)

where \( \tilde{B}_g \) and \( \tilde{R}_g \) equal the numerator of the conditional expectation, referred to as “mean” factor supplies. Using the independent uniform distribution the factor supplies are,

\[
\tilde{B}_g = \left[ \frac{1}{2} \left( B^2 - \left( \frac{A_h}{p_n} \right)^2 \right) \left( \frac{A_h}{p_o} - R_g \right) + \frac{1}{2} B^2 \left( R_g - \frac{A_h}{p_o} \right) - \frac{1}{6} \left( \frac{p_o}{p_n} \right)^2 \left( R_g^3 - \left( \frac{A_h}{p_o} \right)^3 \right) \right] \times \cdots \\
\cdots \frac{1}{(B - B) (R - R)},
\]

and

\[
\tilde{R}_g = \left[ \frac{1}{3} \left( \frac{p_o}{p_n} \right)^2 \left( R_g^3 - \left( \frac{A_h}{p_o} \right)^3 \right) - \frac{1}{2} B \left( R_g^2 - \left( \frac{A_h}{p_o} \right)^2 \right) \right] \frac{1}{(B - B) (R - R)}.
\]

Given the distributions of brawn, that is men’s higher average brawn levels given \( x_m > x_f \), the “mean” brawn supply of men is greater than that of women \( \tilde{R}_m > \tilde{R}_f \),

\[
\frac{\partial \tilde{R}_g}{\partial x} = (R + x) \left( \frac{p_o}{p_n} (R + x) - B \right) \frac{1}{(B - B) (R - R)} > 0,
\]

as long as some agents prefer to work in the old sector, \( p_o R_g > p_n B \). The mean brain supply is greater for women than men,

\[
\frac{\partial \tilde{B}_g}{\partial x} = -\frac{1}{2} \left( \frac{p_o}{p_n} \right)^2 \left( R_g + x \right) - \frac{1}{2} \left( \frac{A_h}{p_o} \right) \left( R_g + x \right) + \frac{1}{(B - B) (R - R)} < 0,
\]

as long as some agents prefer to work in the old sector than stay at home \( p_o R_g > A_h \).

However, more importantly the effect of a rise in the returns to brain-intensive occupations will have a different effect on the average brain supply by gender conditional on the labor force participation, that is

\[
\frac{\partial \tilde{B}_g / LPF_g}{\partial p_n} = \frac{1}{LPF_g^2} \left( \frac{1}{3} \frac{p_o^2}{p_n^3} \left( R_g^3 - \left( \frac{A_h}{p_n} \right)^3 \right) LPF_g + \frac{A_h}{p_n} \left( \frac{A_h}{p_o} - R_g \right) \left( \frac{A_h}{p_n} LPF_g - \tilde{B}_g \right) \right).
\]
All terms are positive, except for the last term \( \frac{A_h}{p_h} LPF_g - \tilde{B}_g \), this term can be positive or negative. Moreover, note that \( LPF_f < LPF_m \) and from above \( \tilde{B}_f > \tilde{B}_m \). Therefore, this last term, which potentially lowers the conditional mean brain supply is smaller for women compared to men given the result suggested above, a rise in the returns to brain-intensive occupations will lower average women’s brain supply compared to men’s. This fall in mean brain can, therefore, potentially slow down the wage gap convergence and even lead to a stagnation in the wage gap.

These results suggest that a model which differentiates between brain-intensive and brawn-intensive jobs, which over time experience a rise in returns to brain due to a shift in labor shares caused by technical change, should be able to capture not only the closing of the employment gap and wage gap observed in the United States since World War II, but also some initial stagnation as observed in the wage gap during the 1960s and 1970s.

5 The United States Simulation

5.1 Calibration

Taking the model to the data requires us to set various household and production parameters, as well as determine the brain and brawn distributions of individuals. In order to do so I match various 1950 data targets.

Production Parameter Estimation

This paper calibrates to the first scenario where the intermediate sectors are a brain-intensive and a brawn-intensive sector. To determine the production parameters \( A_{n,t}, A_{o,t}, \) and \( \phi \), the equivalent regressions of Katz and Murphy (1992, pg. 69) are re-produced, where skilled labor is defined as brain-intensive and unskilled labor is defined as brawn-intensive occupations. Occupations are sorted by their relative brain to brawn inputs such that occupations with \( b > r \) are brain-intensive occupations and occupations with \( b < r \) are brawn-intensive. Full-time workers\(^{10}\) are grouped

\(^{10}\)Full-time workers are workers that worked at least 39 weeks and 35 hrs per week. Prior to the 1976 only hours worked prior to the survey week are recorded.
according to their age group (eight year intervals), sex, education (less than high school, high school, some college, or college), race (white, black or other), marital status (married or single), sector (industry or services) and the type of occupation (brain-intensive or brawn-intensive). I follow Hansen (1993) in estimating efficiency units at time $t$

$$E_t = \sum_k \delta_k L_{t,k},$$

where $L_{t,k}$ is total labor supply of group $k$ and $\delta_k$ is the groups weight, with

$$\delta_k = \frac{\omega_k}{\omega},$$

determined by the average hourly wage of group k over the average hourly wage of the whole population averaged over time and individuals. The resulting relative unit wage of brain over brawn and relative labor supply of efficiency units is shown in figure 2, section 1. Taking the natural logarithm of (9), and assuming, as in Krusell et al. (1997), a log-linear brain-biased technical change over time, i.e. $\ln \frac{A_{n,t}}{A_{o,t}} = \zeta_0 + \zeta_1 t + \eta_t$, leads to the following regression estimation,

$$\ln \left( \frac{p_{n,t}}{p_{o,t}} \right) = a_0 + a_1 t + a_2 \ln \left( \frac{L_{n,t}}{L_{o,t}} \right),$$

where $a_0 = \ln \left( \frac{\lambda_n}{\lambda_o} \right) + \phi \zeta_0$, $a_1 = \phi \zeta_1$, $a_2 = \phi - 1$ and $\frac{L_{n,t}}{L_{o,t}}$ are the relative efficiency units of brain-intensive to brawn-intensive occupations. Table A.3 in appendix A provides the regression estimates, normalizing $\zeta_0 = 0$ the resulting parameter values of the annual growth rate of brain-biased technical change $\gamma_o - \gamma_n$ and the substitution parameter $\phi$ are summarized in table 2, while $\frac{A_{n,t}}{A_{o,t}}$ is normalized to one in 1950. $\lambda_n$ is set to match the 1950 labor share of brain-intensive occupations in the economy, which is about 40.3 percent. Lastly, the share parameters $\alpha_n$ and $\alpha_o$ are matched to the brawn deviations in 1950 in each sector (see figure 7) together with the remainder of the parameters determining the distribution of brain and brawn (see below). Note, the fairly steady brain and brawn deviations over time, suggesting the grouping of occupations to be fairly robust over time and a brain-biased technical change to be able to capture most of the
change in aggregate brain and brawn supply deviations from figure 4.

Table 2 summarizes the main production parameters estimates,

<table>
<thead>
<tr>
<th>Production Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Substitution parameter $\phi$</td>
<td>0.6032</td>
</tr>
<tr>
<td>Share of the new sector $\lambda_n$</td>
<td>0.51</td>
</tr>
<tr>
<td>Difference in annual relative TFP growth rate $\gamma_n - \gamma_o$</td>
<td>0.0147</td>
</tr>
</tbody>
</table>

### Agents’ Ability

Brain and brawn are assumed to be joint normally distributed with correlation $\rho$. This requires us to estimate a total of six parameters; the mean, $\mu_b$, and standard deviation of brain, $\sigma_b$, and the two means $\mu_{r,m}, \mu_{r,f}$, the standard deviation of brawn, $\sigma_r$, and the correlation $\rho$. In addition to matching $\alpha_n$ and $\alpha_o$ from above, I match the following data targets,

1. Proportion of working men in the new sector in 1950;
2. Proportion of working women in the new sector in 1950;
3. Male Brawn deviations in 1950;
4. Female Brawn deviations in 1950;
5. New Sector Brain deviations in 1950;
6. New Sector Brawn deviations in 1950;
7. Old Sector Brain deviations in 1950; and

To determine the four-dimensional joint brain and brawn distributions of married couples, I assume perfect assortive matching in brain abilities $\rho_b = 1$. Therefore, only men and women of the same
brain level marry, with no restrictions on brawn matching, which results in mostly female secondary wage workers.

**Remaining Baseline Parameters**

Following Jones et al. (2003), I set the fraction of married households to 0.6. The consumption substitution parameter $\epsilon_\nu = 1.8$ is set given the estimates of other studies where estimates range from 1.3 in McGrattan et al. (1997), 1.8 in Rupert et al. (1995), 1.85 in Aguiar and Hurst (2006) to 2.3 in Chang and Schorfheide (2003). Lastly, since the labor choice is a zero/one decision, the model is unable to calibrate both single and married women’s labor force participation with a common household productivity parameter, similarly, Rupert et al. (1995) find different elasticity estimates for married and single households in a continuous labor choice model. However, the estimates on married household production have large standard errors. Therefore, in the calibration of all marital status (I also calibrate the model for married households exclusively) I will allow household productivity to differ across married and single households and these two parameters will be determined by matching married women’s 1950 labor force participation and single women’s 1950 labor force participation, respectively.

### 5.2 Simulation of Technical Change, Employment & Gender Wage Gap

Selected initial results on a calibration of married households exclusively are presented first. When the difference in annual relative TFP growth rate $\gamma_n - \gamma_o = 0.0147$, as obtained from the regression estimates is used, the calibrated model only produces 62.9 percent of the rise in the new sector labor share and is able to explain about 38 percent of the rise in married female labor force participation observed in the data. Adjusting this growth rate to $\gamma_n - \gamma_o = 0.025$ matches the rise in labor shares of the new sector perfectly. This adjusted model produces the following results on the married female labor force participation and the wage gap:

1. Married women’s labor force participation rises by 31 percentage points (see figure 8), meaning the model explains 67 percent of the rise in married female labor force participation; and

2. The wage gap closes by 8 percentage points from 1950 to 2005 (see figure 9), equivalent to
62 percent of the close in the data.

While, in the current calibration married men’s labor force participation remains fairly constant a fall can be achieved by relaxing the assumption of perfect assortive matching (an assumption made due to the additional computational burden). As a consequence, some men would cease to be primary worker and therefore drop out of the labor market as their returns to brawn would diminish, while their wifes would become the primary worker, increasing married women’s overall rise in labor force participation further.

On the wage gap, the model fails in certain aspects. While it is able to generate a 8 percentage point rise from 1950 to 2005, it fails to match the initial wage discrepancy between men and women. The model generates a wage gap that is 18.7 percentage points higher in 1950 (note the scale difference on the left and right y-axis in figure 9). The reason is that married women only work in the labor market if their wages are very attractive given their predominant secondary earner status in the household and substitution of market and home produced goods. Therefore, only married women with high wage potential enter the labor market in the 1950s resulting in a small wage gap. The model does generate a slightly slower growth rate of average female wages in the beginning of the period due to women with lower “ability” levels entering the labor market, which generates a wedge in average factor supplies of new entrants versus existing workers. However, the stagnation far from matches the data, suggesting that other factors such as a wedge of work experience between workers and new entrants could have played a substantial role in the stagnation of average wages.

The self-selection of high earning women becomes further apparent when analyzing brain and brawn standard deviations from the mean by gender over time (see figure 10). Note, that women’s brain supply in 1950 is considerable higher in the model than the data generating part of the excessively high average female wage. While men’s brain supply is too low in the model. The deviations of female brawn supplies are well matched over time, while the fall in men’s brawn supplies and the rise in brain supplies are too slow in the model economy.

Lastly, the model is able to replicate most of the labor share trends for men and women by sector (see figure 11), but overestimates the initial proportion of women and underestimates the
initial proportion of men in the new sector. The brain and brawn deviations from the mean by sector are also well matched over time (see figure 12), except for the rise in brain supplies within the new sector and the fall in brawn supplies in the old sector, which follow from the assumption on constant brain and brawn productivity within each sector, i.e. constant $\alpha_j$’s.

Repeating this exercise to include single men and women leads to the following results:

1. Married women’s labor force participation rises by 17 percentage points (see figure 13), meaning the model explains 37 percent of the rise in married female labor force participation (similar to the results with the original TFP growth rates of 0.0147 when omitting single households);

2. Single women’s labor force participation rises by 13 percentage points (see figure 13), almost entirely matching the trend observed in the data (89 percent);

3. Both married and single men’s labor force participation rates are fairly constant, however, the model overestimates the proportion of working single men and underestimates the proportion of married working men (see figure 13);

4. The wage gap closes again by 8 percentage points from 1950 to 2005 (see figure 14), equivalent to 62 percent of the close in the data. However, the model is unable to generate a prolonged period of stagnated wages (or slower growth in the wage gap), with the rise in the relative returns to brain outweighing the slower growth in female brain supply now almost entirely.

Note that all other results are almost identical to the model without single households and are therefore omitted here.

6 Extension: Endogenous Brain-Biased Technical Change

This study has modeled brain-biased technical change as an exogenous process and divided the economy into two sectors, a brain-intensive and brawn-intensive sector that are substitutes in the production of the aggregate good. The next research step is to endogenize this process, i.e. the
coefficients on brain productivity, \( \alpha_j \), in a two sector model of the industrial and service sector, should be allowed to endogenously change over time providing a theory of the origin and evolution of brain-biased technical change. One possible way to endogenize this is to assume that WWII took most of the brawn supply in the economy away with men leaving for the war. Assuming that firms can invest some fraction of labor toward innovation, efforts in innovation will shift heavily toward brain-intensive production favoring women’s comparative advantage. Given the complementarity of industrial goods and services, some time will be spend on innovating the production process in both sector toward a more brain-biased production.

Decomposing brain and brawn deviations by the industrial and service sector\(^{11}\) (see figure 15) illustrates this brain-biased technical change within both sectors since World War II. Note that (1) the industrial sector requires above average brawn, while the service sector requires below average brawn, (2) brain requirements rise continuously in both the industrial and service sector, with consistently greater levels in the brain sector, (3) brawn requirements fall sharply from the 1950s to 1970s in the industrial sector. Point (2) and (3) can be interpreted as within sector brain-biased technical.

In summary, as theory that allows for endogenous innovation playing to the strength of the economy should be able to explain some the trends in brain-biased technological change observed in figure 15 for both sectors.

7 Conclusion

The purpose of this study is to assess the importance of labor demand changes on women’s labor force participation and wages. The focus is on changes in brain and brawn input requirements in the aggregate production process. A considerable rise in brain and fall in brawn requirements is documented from the 1977 DOT. Preliminary time trends and wage regression estimates suggest

\(^{11}\)I use the classification by Fuchs (1977), where the industrial sector includes agriculture, fishing, forestry, mining, construction, manufacturing, and transportation, communication and other utilities, while the service sector includes all services and the government sector. Transportation, communication and other utilities, are contrary to Buera and Kaboski (2006) grouped with the industrial sector as they have similar labor demand requirements as the industrial sector. However, the results do not change substantially with this classification as this subsector is relatively small. Note that, while Ngai and Pissarides (2005) separate primary from secondary industries, they group transportation, communication and other utilities with the industrial sector as well.
these labor demand changes to have had a sizable impact. For example, a fall in brawn prices and brain supplies can explain about 30 percent of the evolution of the wage gap since the 1950s in a Mincer-type wage regression. The simulation of the general equilibrium model provides further insight of the dynamics of such a labor demand change and the possible quantitative effect of these changes on women’s labor force participation and the closing wage gap. Initial results on married households alone shows that brain-biased technical change is able to replicate about 67 percent in the rise in married female labor force participation and about 62 percent in the closing wage gap, with an initially slower growth rate in average female to male wages due to selection bias. Results of a model including single households lower the rise of married female labor force participation by slightly to 37 percent, while the percent explained by the model of the closing wage gap remains at 62 percent. In addition, the model with single households is able to explain most of the rise in single women’s labor force participation, that is 89 percent of the rise in the data can be explained by the model. Clearly, adding other features that may generate some non-linearity (brain-biased technical change is by assumption linear) such as experience accumulation or educational choice, creating a bigger wedge between working women and new entrants, may help match the initial wage gap and the fall from 1950 to 1980 as well.

Lastly, this study has modeled brain-biased technical change as an exogenous process, but the next research step is to endogenize this process as roughly outlined in section 6.
## Appendix A: Regression Estimates

Table A.1: *Factor Loading Estimates* ($\Lambda$)

<table>
<thead>
<tr>
<th>Job Characteristic</th>
<th>Coefficient ($\Lambda_i$)</th>
<th>Job Characteristic</th>
<th>Coefficient ($\Lambda_i$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Directing/Controlling</td>
<td></td>
</tr>
<tr>
<td>Brawn Factor</td>
<td></td>
<td>Interpreting Feelings/Ideas/Facts</td>
<td></td>
</tr>
<tr>
<td>Repetitive Work</td>
<td>0.30406</td>
<td>Influencing People</td>
<td>0.37265</td>
</tr>
<tr>
<td>Climbing/Balancing</td>
<td>0.77651</td>
<td>Making Evaluations Based on Judgment</td>
<td>0.60055</td>
</tr>
<tr>
<td>Stooping/Kneeling/Crouching/Crawling</td>
<td>0.81000</td>
<td>Making Judgments/Decisions</td>
<td>0.43480</td>
</tr>
<tr>
<td>Strength Requirement</td>
<td>0.88075</td>
<td>Dealing with People</td>
<td>0.49332</td>
</tr>
<tr>
<td>Environmental Exposure$^a$</td>
<td>0.77673</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Indoor or Outdoor Work</td>
<td>0.68110</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Brain Factor</td>
<td></td>
<td>Coordination Factor</td>
<td></td>
</tr>
<tr>
<td>Reasoning Development</td>
<td>0.96668</td>
<td>Seeing</td>
<td>0.77650</td>
</tr>
<tr>
<td>Mathematical Development</td>
<td>0.89217</td>
<td>Spatial Aptitude</td>
<td>0.43418</td>
</tr>
<tr>
<td>Language Development</td>
<td>0.95275</td>
<td>Form Perception</td>
<td>0.71349</td>
</tr>
<tr>
<td>Specific Vocational Preparation</td>
<td>0.77567</td>
<td>Motor Coordination</td>
<td>0.84869</td>
</tr>
<tr>
<td>General Intelligence</td>
<td>0.94685</td>
<td>Finger Dexterity</td>
<td>0.88302</td>
</tr>
<tr>
<td>Verbal Aptitude</td>
<td>0.94068</td>
<td>Manual Dexterity</td>
<td>0.66313</td>
</tr>
<tr>
<td>Numerical Aptitude</td>
<td>0.83968</td>
<td>Eye-Hand-Foot Coordination</td>
<td>0.07607</td>
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<tr>
<td>Clerical Aptitude</td>
<td>0.70447</td>
<td>Color Discrimination</td>
<td>0.37763</td>
</tr>
<tr>
<td>Talking and Hearing</td>
<td>0.57950</td>
<td>Attaining Precise Tolerances</td>
<td>0.72865</td>
</tr>
<tr>
<td>Performs Variety of Duties</td>
<td>0.24961</td>
<td>Reaching/Handling/Fingering/Feeling</td>
<td>0.50627</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Making Decisions based on Measurable Criteria</td>
<td>0.30894</td>
</tr>
</tbody>
</table>

Notes: Estimated using maximum-likelihood factor analysis.

$^a$Environmental conditions such as the presence of heat, cold, humidity were combined to one variable prior to the estimation.
Table A.2: *Factor Price Estimates* \((p_b, p_r)\)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Brain</td>
<td>0.1138981*</td>
<td>0.153931*</td>
</tr>
<tr>
<td></td>
<td>(0.0002042)</td>
<td>(0.0000898)</td>
</tr>
<tr>
<td>Brawn</td>
<td>0.0446319*</td>
<td>0.13126*</td>
</tr>
<tr>
<td></td>
<td>(0.0001997)</td>
<td>(0.0000963)</td>
</tr>
<tr>
<td>Brain × T</td>
<td>0.0629789*</td>
<td>0.0555917*</td>
</tr>
<tr>
<td></td>
<td>(0.0002157)</td>
<td>(0.0001103)</td>
</tr>
<tr>
<td>Brawn × T</td>
<td>0.0793846*</td>
<td>-0.0600552*</td>
</tr>
<tr>
<td></td>
<td>(0.0002184)</td>
<td>(0.0001233)</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.3170</td>
<td>0.2576</td>
</tr>
</tbody>
</table>

Notes: * Statistically significant at the 1 percent confidence level. Standard Errors are in parenthesis. The regression also includes controls for age, age squared, years of education, marital status, race, region coordination factor, and a T-year dummy.

Table A.3: *Production Regression Estimates*

<p>| | |</p>
<table>
<thead>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>0.0416971</td>
</tr>
<tr>
<td></td>
<td>(0.0393867)</td>
</tr>
<tr>
<td>Time Trend</td>
<td>0.0088655*</td>
</tr>
<tr>
<td></td>
<td>(0.000657)</td>
</tr>
<tr>
<td>Brain to Brawn Labor</td>
<td>-0.3967528*</td>
</tr>
<tr>
<td></td>
<td>(0.0593553)</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.739</td>
</tr>
</tbody>
</table>

Notes: * Statistically significant at the 1 percent confidence level. Robust standard Errors are in parenthesis.
Appendix B: Married Women’s Labor Supply

It is necessary to determine the appropriate margin in the contribution of the rise of female participation, that is, was it due to the increase in hours worked by working women (intensive margin), was it due to new entrants to the labor market (extensive margin) or possibly both. We see a tremendous rise in labor force participation in figure 1. Moreover, the growth in female labor force participation has mostly been on the extensive margin, i.e. labor force participation rose while average weekly hours worked of the working population remained fairly constant (with a slight convex downward trend from 1950 to 2005, see figure C.1). For example, single employed women worked nearly 40 hours/week in 1950 and slightly less than 40 hours/week in 2005, while married women worked about 38 hours/week.
References


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Figure 1: Wage Gap and Female Labor Force Participation

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Figure 10: Brain and Brawn Deviations by Gender - Model vs. Data
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Figure 12: Brain and Brawn Deviations by Sector - Model vs. Data
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Extension

Figure 15: Deviation of Labor Input Supply by Industry

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Figure B.1: Average Weekly Hours Worked by Gender and Marital Status